Distributed Adaptive Consensus for Multi-Agent Systems Subject to Uncertainties

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Statement of Originality

I hereby certify that the work embodied in the thesis is my own work, conducted under normal supervision. The thesis contains no material which has been accepted, or is being examined, for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made. I give consent to the final version of my thesis being made available worldwide when deposited in the University's Digital Repository, subject to the provisions of the Copyright Act 1968 and any approved embargo.

Imil Hamda Imran

Acknowledgement of Authorship

I hereby certify that the work embodied in this thesis contains submitted papers of which I am a joint author. I have included as part of the thesis a written declaration endorsed in writing by my supervisors, Prof. Minyue Fu and Prof. Zhiyong Chen, attesting to my contribution to the joint publications.

By signing below I confirm that Imil Hamda Imran contributed towards idea generation, theoretical development and other works for the publications as listed in Section 6.3.

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Contents

List of Figures

Abstract

Research on cooperative control of multi-agent systems has drawn increased attention from control engineers in recent decades. Inspired by natural phenomena, this research has been developed to become more practical and reliable in implementation. Consensus is one of the most active and very crucial research topics in cooperative control of multi-agent systems. One of the unavoidable problems in developing consensus control for multi-agent systems is the presence of uncertainties in the dynamic models. Adaptive control is a research line applied to solve consensus problems for multi-agent systems subject to uncertainties. In this thesis, we establish a distributed adaptive consensus framework for multi-agent systems with uncertain dynamics.

There are two main problems in designing distributed adaptive consensus control for general multi-agent systems. First, the adaptive law cannot always be implemented in a distributed fashion because it depends on the gradient of a (centrally constructed) Lyapunov function. Consequently, distributed adaptive consensus can only be applied for limited cases. In this thesis, we establish a distributed adaptive consensus framework to overcome this problem by proposing a novel distributed adaptive scheme that does not rely on the gradient of a Lyapunov function. An application of our framework is presented to solve the consensus problem in second-order multi-agent systems under a directed topology.

The second problem is the presence of nonlinearly parameterized dynamics in multi-agent systems. It is always difficult to handle nonlinearly parameterized uncertainties in adaptive control. Some results have been obtained for special cases. In addition, none of the existing results are applicable to

networked systems with nonlinearly parameterized dynamics. In this thesis, we develop a distributed adaptive framework for multi-agent systems subject to nonlinearly parameterized uncertainties. The linear parameterization assumption is removed by proposing a novel distributed adaptive update law. Therefore, our scheme is more applicable to general nonlinear multi-agent systems. A specific implementation of our framework is presented for nonlinear second-order multi-agent systems. To illustrate our approaches, we present some numerical examples and simulations with various settings.

1

Introduction

This thesis studies the distributed adaptive consensus framework for multi-agent systems (MASs) that can be implemented in many applications such as consensus of Unmanned Aerial Vehicles (UAVs), consensus of mobile robots, synchronization of load balancing and so on. The objective of our study is to establish some general frameworks to maintain the cooperative motion of individual agents in MASs, where the control protocol can be applied to both linear MASs and nonlinear MASs subject to uncertainties.

In this chapter, we present an introduction to MASs, cooperative control of MASs and adaptive control in the first section. Subsequently, we present the literature review of MASs with linear and nonlinear dynamics. Then we explain the main research outcomes of this thesis and its contributions by comparing our work with existing research. The outline and organization of the thesis are presented in the last section.

1.1 Background and Motivation

1.1.1 Multi-Agent Systems

The collective movement of animals in a social group is an important natural phenomenon. Individually, each animal has its own pattern and motion. Nevertheless, the collective motion depicts the group as one entity with some desirable global or collective responses to external influences. The aggregate dynamics aim towards achieving objectives such as migration, foraging and protecting the group from predators. The synchronized movement and responsive movement bring about a choreographic movement as in a dance, yet this displayed pattern is not of a planned script, but the result of instantaneous responses and decisions by individual members.

Figure 1.1: A group of birds flying in V-formation [1]

Figure 1.2: A school of fish [2]

A group of animals moving together allow the group to achieve what the individual is unable to do. For example, a group of birds can migrate to a distant site by flying in V-formation as illustrated in Fig. 1.1. In this situation, the energy required by an individual bird to fly can be reduced by remaining in the wingtip vortex up-wash of those ahead. Therefore, the weak and young birds can also survive the migration. Another example is a school of fish as illustrated in Fig. 1.2. The predators have more difficulties to catch an individual fish moving in a social group because a group of fish

Figure 1.3: A herd of horses [3]

can scatter and cluster quickly. To elude predators, a herd of horses (Fig. 1.3) move together in an organized way to a distant site foraging for food. This type of behaviour can also be seen in a group of humans in a panic and mob scenario, where there is tremendous pressure to align with the group's collective leader, rather than following individual dispositions.

The flow of information has a crucial role in the movement of a group of animals, but each individual is aware only of its neighbour's motions. The information obtained by the individual animals affect their own movements, which influences the movement of the group overall. Different information flows and different species generate different types of motions. For example, the formation flight of birds illustrated in Fig. 1.1 indicates that an individual bird is only concerned about the movement of a few neighbours beside it, however, the information flow allows all the birds fly in a particular formation. In another example, such as flocking of horses, besides the motion of a few neighbours, the vibration of the earth may influence the motion of individual horses that may not be even close by.

Illustrating the collective behaviour of animal groups using a computer simulation has been a challenge for researchers in recent decades. It is impossible to script the movement of each individual in a planned trajectory. On the other hand, analyzing a group hinged on social behaviour is actually complex, yet, the individual in a collective movement seems to follow simple rules that make their movement efficiently responsive and practically instantaneous. The collective movement of animal groups can be simulated in a computer by bequeathing each individual with the same set of rules, which allows it to respond to the situation encountered. The collective movement of the group is the accumulated responses of the individual animals.

The collective motion of animals has drawn increased attention to the study of animal behaviour in nature. The aggregate motion of flock birds was simulated using computer animation in [4]. In this animation, a distributed behaviour of each bird was analyzed, where each flying bird worked independently to keep together and avoid collisions with its neighbour and with any object in their environment. Inspired by biological interaction, self-ordered motion was studied in [5] by investigating phase transition in a system of self-driven particles. Some observations and results of the animal behaviour with the essential aspects of collective motion can be found in [6, 7].

This natural phenomenon and some preliminary results presented by researchers in distributed computing and collective biological motion have inspired the control engineers and theoreticians to develop control protocols for MASs. A MAS is comprised of multiple interacting intelligent agents in a networked environment. Each agent has characteristics like interactivity and autonomy. The connected agents have the capability to respond to the environment based on the information received from a network. An individual agent is represented by some physical entity such as a satellite, a robot manipulator, an unmanned aerial vehicle (UAV), and various other types of robots.

MAS is a broad field of research and relates to knowledge in mathematics, biology, physics, robotics, control automation and so on. In control automation research, MASs are used in many applications such as cooperative control of unmanned aerial vehicles (UAVs) [8, 9, 10, 11, 12, 13, 14], cooperative control of mobile robots [15, 16, 17, 18, 19, 20, 21, 22], rendezvous and proximity operations of satellites [23] and so on.

1.1.2 Cooperative Control of MASs

Research problems in cooperative control of MASs have been widely studied in various aspects such as flocking, synchronization/consensus, formation control, obstacle avoidance, swarming and so on. The main objective of cooperative control is that the state of every agent is required to reach a particular agreement on some physical quantities of interest such as position, velocity, temperature, voltage, force, attitude and so on.

Flocking control is one of the research problems in control automation studying a collective behaviour of a group of agents in a particular connected environment. The mechanism in flocking is repulsive and attractive control actions. This mechanism was described in [4, 24] as the three basic rules in flocking:(1) Flock centring, where each agent keeps close to their nearby mates, (2) Alignment, where each agent adjusts to match their velocity to their nearby mates, (3) Avoiding collision, where each agent avoids collisions with the nearby mates. Some results in flocking control of MASs with different settings can be found in [22, 24, 25, 26, 27, 28].

Consensus is another fundamental research problem in cooperative control of MASs. The main goal in consensus is to achieve an agreement in a particular physical quantity. The dynamic behaviour of each agent in consensus is represented by their state. That is, consensus requires that the state of every agent reaches an agreement in some particular sense. A control protocol is designed based on local information and information exchange between neighbouring agents. There are two general scenarios in consensus: leader-following and leaderless. In the leader-following setting, the followers are required to follow one or more leaders.

One of the applications of consensus is formation control. In formation control, every agent is required to follow a desired configuration or formation. Generally speaking, the displacement and distance-based measurements are commonly used for consensus-based formation control. There are two common approaches for consensus of MASs. The first is a centralized control approach, where control protocols for every agent are designed by a central station based on the information of all the agents. The second approach is distributed control. In this approach, every agent is governed by some distributed control protocol using local information. The local control is generated for each agent based on the relative measurements with its neighbours in the network. Compared with the centralized control, the distributed control has several advantages as it is more efficient, more flexible and cheaper to be applied. In this thesis, our focus is to study consensus protocols of MASs that can be implemented in a distributed fashion.

1.1.3 Adaptive Control

One of the main issues in control automation is the presence of uncertainties in the system dynamics due to various disturbances such as structural damage, the change of dynamics and the change of environment in time. As a result, the system model may have uncertain dynamics, unknown and time-varying parameters that can degrade the control performance.

The aforementioned situations are challenging for control design. A fixed gain feedback control has some degree of robustness to handle uncertainties in the system dynamics. The bounds of uncertainties are required to be known as *prior* information in control design. The control gain is fixed and tuned to the worst case rather than to actual physical systems. The stability is guaranteed if the uncertainties are within the bounds.

Another approach to handling uncertainties is adaptive control. It is a control strategy that has the ability to adjust itself to handle the changing environment under unforeseen and adverse conditions. In this approach, *prior* information about the bounds of uncertainties is not required. Also, the controllers are not required to be adjusted for the worst case. The basic idea of adaptive control is parameter estimation, which is generated by adaptation laws. The estimation parameters become inputs for the controller to improve it. A proper adaptation law is required to be designed to learn the changing parameters by processing the output of the systems. Therefore, new parameter estimation and a new control gain can be used to achieve the desired performance.

1.2 Literature Review

1.2.1 Consensus of Linear MASs

Control of MASs is motivated by collective phenomena in natural systems and extensive engineering applications, including multiple UAVs and mobile robots, distributed sensor networks, load balancing and so on. Consensus is one of the most active research topics in MASs from the systems and control perspective and it has achieved rapid progress in recent years [29]. The goal is to design collective algorithms for a group of agents such that they achieve certain agreement.

Consensus control has been intensively developed from MASs with linear to nonlinear dynamics, from homogeneous to heterogeneous systems, from ideal communication network to communication constraints, from a leader-following case to a leaderless case, from theory to practice and so on. Nevertheless, there are still many open problems that need more attention by control engineers.

From the viewpoint of the system structure, MASs can be classified into linear and nonlinear systems. At the beginning of the research, consensus control was studied for linear MASs. Many control strategies with interesting results have been developed under this setting. MAS with linear dynamics is commonly represented by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i + B_i u_i \\ y_i(t) &= C_i x_i(t),\ i = 1, \cdots, n, \end{aligned} \tag{1.1}$$

where $x_i \in \mathbb{R}^l$ is the state, $u_i \in \mathbb{R}^m$ is the control input and $y_i \in \mathbb{R}^q$ is the output of agent i with constant matrices A_i, B_i and C_i.

Early research works were mostly for MASs consisting of first-order or single integrator dynamics [30, 31, 32]. The general solution for consensus in these MASs was presented in [30]. The introduction to linear first-order MASs and sufficient conditions to achieve consensus were provided in [33, 34].

The dynamics of linear first-order MASs with n agents is commonly given by

$$\dot{x}_i(t) = u_i(t). \tag{1.2}$$

Consensus is reached when the states of all agents converge to a common constant value. For the leaderless case, consensus is usually said to be achieved if

$$\lim_{t\to\infty} x_i(t) - x_j(t) = 0. \tag{1.3}$$

The common control protocol for MASs (1.2) to achieve consensus (1.3) for the leaderless case [30, 31, 33, 34, 35, 36] is

$$u_i = \sum_{j=1}^{n} a_{ij}(t)(x_j(t) - x_i(t)), \tag{1.4}$$

where $a_{ij}(t)$ is the edge from agent j to i. The control protocol (1.4) can be applied to achieve consensus under some constraints on the network.

In many practical applications such as vehicles, the dynamics of each agent contains position and velocity as the state. This means that the system dynamics is in the form of second-order or double integrator. Corresponding to general dynamics (1.1), the dynamics of linear second-order MASs is commonly represented when $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$,

$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C_i = I_2$. Or in other words

$$\begin{aligned} \dot{x}_{1_i}(t) &= x_{2_i}(t) \\ \dot{x}_{2_i}(t) &= u_i(t), \end{aligned} \tag{1.5}$$

where the states x_{1_i} and x_{2_i} represent the position and velocity of the system respectively and u_i is the control input of agent i.

Compared with first-order linear MASs, the second-order systems are more complicated for consensus control. The stability analysis in first-order MASs cannot be simply extended to second-order systems. One of the common solutions for the leaderless case with a strongly connected network topology is

$$u_i\,(t) = k_1 \left(\sum_{j=1}^{n} a_{ij}(t)(x_{1_j}(t) - x_{1_i}(t)) + k_2 \sum_{j=1}^{n} a_{ij}(t)(x_{2_j}(t) - x_{2_i}(t)) \right) \tag{1.6}$$

for some k_1 and k_2. Some control protocols have been developed to find the sufficient condition for the gain k_1 and k_2 [33, 34, 36].

Many results were obtained for linear MASs with second-order dynamics under different settings [32, 33, 34, 36, 37, 38, 39, 40]. An introduction and convergence analysis for general linear MASs including first and second-order MASs was presented in [33, 34, 36]. A leader-following consensus control for linear second-order MASs under a time-varying topology was studied in [37]. Consensus control was developed in [32] for second-order linear MASs with some restrictions in communication. In [36, 38], the consensus protocol and its conditions were studied under a fixed and switching topology. Consensus of MASs with double-integrator dynamics was developed in [39] without relative velocity measurements. Some necessary and sufficient conditions for consensus of second-order MASs with linear dynamics was investigated in [40].

Leaderless and leader-following consensus for first and second-order MAS with input delay were proposed in [41] for directed graphs based on Lyapunov and Nyquist stability approaches. In [42], consensus of second-order MASs with linear dynamics and time-delay was improved using a weighted average prediction. Another consensus for second-order MASs with time-delay can be found in [43]. In [44, 45], the control protocol was proposed for consensus of second-order linear MASs with communication delay and switching topology. LMI was studied in [46] for leaderless consensus of a class of second-order linear MASs.

Consensus of linear MASs with higher-order dynamics is another challenging problem. The stability analysis in first and second-order MASs cannot be simply extended to higher-order systems. Some interesting results have been obtained, for example in [47, 48, 49]. The consensus control for higher-order MASs with identical multiple-input and multiple-output (MIMO) linear dynamics was developed in [47] using output feedback controllers. In [48], consensus control for higher-order linear MASs was studied under fixed and switching topologies. A consensus protocol for higher-order MASs with linear dynamics was proposed in [49] under switching topology and occasionally missing control inputs.

Research interest in linear MASs also shifts from homogeneous to heterogeneous MASs. Corresponding to systems (1.1), MASs can be said to be homogeneous or identical when $A_i = A$, $B_i = B$ and $C_i = C$ for all i. Some results on MASs with homogeneous dynamics can be found in [49, 50, 51]. Leaderless consensus of MASs with heterogeneous dynamics is more complicated. In [52, 53], a control protocol for linear heterogeneous MASs was developed by synchronizing the output of every agent to a local reference model. A dynamic controller was proposed in [54] for output consensus of heterogeneous MASs with uncertainties.

Another major research line on consensus of linear MASs is optimal control. Some interesting results on optimal consensus have been released under various settings. Stochastic linear quadratic regulators (LQRs) were proposed in [55, 56, 57] with indefinite control weight cost for optimal consensus of MASs. An optimal leader-following consensus for linear MASs with identical dynamics under a fixed network was proposed in [58] using LQR, observer design and output feedback control. LQR was also proposed in [59] for coupled stabilizable MASs in homogeneous dynamics under a fixed directed topology. Sub-optimal hierarchical feedback control for leader-following consensus for linear homogeneous MASs is studied using LQR in [60]. Necessary and sufficient conditions for globally optimal consensus of homogeneous linear MASs was studied in [61]. In [62], iterative adaptive dynamic programming was developed for optimal leader-following consensus of nonidentical linear MASs. An optimal leaderless consensus of identical linear MASs under a fixed directed network can be found in [63]. Valuable information about Lyapunov, adaptive and optimal cooperative control design under directed communication graphs can also be found in [64].

1.2.2 Consensus of MASs with Nonlinear Dynamics

MASs with linear identical dynamics is only a special case. The system dynamics of MASs can be nonlinear in many practical cases. Therefore, the aforementioned control strategies for consensus of linear MASs as described in Subsection 1.2.1 cannot be applied anymore. The convergence analysis of consensus under this situation becomes more complicated. Although not as much research has been done on MASs with linear dynamics, some interesting results have been obtained for consensus of nonlinear MASs.

At the beginning of the research, the consensus controllers were designed for MASs with simple nonlinear dynamics, which satisfy Lipschitz conditions. In several cases, linear control protocols are still able to guarantee consensus of nonlinear MASs satisfying Lipschitz conditions. Examples include [40] for first-order MASs, [65, 66] for second-order MASs and [67] for more general MASs. Consensus control was studied in [68] for leaderless MASs with Lipschitz nonlinear dynamics under a switching topology. An adaptive control protocol was designed in [67] without global information for MASs with general linear and Lipschitz nonlinear dynamics. Some interesting results on consensus of MASs with non-Lipschitz nonlinear dynamics can also be found in [69, 70, 71, 72, 73, 74, 75] under various settings.

In many practical situations, the agent dynamics are usually subject to uncertainties that also induce heterogeneity. To handle system uncertainties, an internal model based approach has been proven to be effective. For example, linear internal model based consensus techniques can found in [53, 54, 76, 77] in different settings. The basic idea is to introduce a reference trajectory for each agent and collectively synchronize these references and hence agent outputs.

While certain nonlinearities of agent dynamics may be handled by feedforward compensation, see, e.g., [78], uncertain nonlinearities likely bring more technical challenges. Most existing results are based on internal model design. For instance, in [79], the authors designed controllers for MASs of second-order nonlinear dynamics with agreement on a constant. More general nonlinear dynamics were studied in [80, 81] that require that all agents exchange full state information. The most sophisticated result was given in [82] in the output communication setting using a small gain theorem. Other relevant internal model designs can be found for cooperative output regulation in a leader-following setting; see, e.g. [83, 84]. Robust consensus protocol of second-order nonlinear heterogeneous MAS with communication delay can be found in [85, 86].

Another research line is to deal with system uncertainties, in particular, unknown parameters, using adaptive control. As with traditional adaptive control, the certainty equivalence principle has a two-step design scheme. First, a controller is developed for the systems with an ideal situation, where the uncertain parameter is assumed to be known and it renders a Lyapunov function. Then in the second step, the uncertain parameters in the controller are replaced by their estimates, which are updated by an adaptive law along the gradient of a suitable Lyapunov function.

In the literature, such an adaptive control scheme has been investigated for MASs in some situations. For example, a first-order MAS was studied in [87] for an undirected graph. The result was presented in a more general framework in [88]. A similar adaptive technique was used in [89] for both first and second-order MASs with a Nussbaum gain added to deal with unknown control directions. Pinning consensus was proposed by adaptively tuning the coupling strength in [90]. Another result on pinning consensus control can be found in [91]. The adaptive consensus was studied in [92] for undirected graph. Also, for undirected time-varying graphs under the jointly connected condition, an adaptive scheme was studied for first-order MASs in [93] and [94] for the leader-following and leaderless settings, respectively. In particular, in [93] each agent requires "not only the information of its neighbours but also the information of its neighbours' neighbours" and then in [94] the approach was improved to a purely distributed design.

Consensus control for MAS becomes significantly complicated under a directed topology due to the associated asymmetric Laplacian matrix. Some interesting results on leader-following consensus of MASs with the unknown nonlinearities under a fixed directed network can be found for first-order in [95], second-order [96] and higher-order [97]. The nonlinear dynamics with the unknown nonlinearities in [95, 96, 97] were approximated by neural network (NN). It is noted that consensus of MASs is achieved with a residual error, not asymptotically exact.

Adaptive control is a common approach used to estimate the unknown constant parameters in the system dynamics. Many existing methods have studied the effectiveness of adaptive control to guarantee the stability for a single agent or system. However, most of the existing results are under an essential assumption that the nonlinear dynamics is in the class of linearly parameterized nonlinear systems. The nonlinear dynamics is called linearly parameterized if the unknown constant parameters appear linearly in the nonlinear function. For instance, the model reference adaptive control (MRAC) method was proposed in [98, 99, 100] to handle systems with linearly parameterized nonlinear dynamics. In [101, 102, 103, 104, 105, 106], L_1 adaptive control, which is an extension of MRAC, was developed for systems with uncertain nonlinear dynamics, but still in the class of linearly parameterized nonlinear models.

Adaptive control under linearly parameterized assumption is unable to be widely applied for prevalent applications in many nonlinearly parameterized dynamics models. The system is called a nonlinearly parameterized model when the unknown constant parameters appear nonlinearly in the nonlinear dynamics. The system dynamics with the nonlinearly parameterized model are inevitable in industrial applications, for example, fermentation processes [107], distillation columns, chemical reactors, separation processes and bioreactors [108].

Nonlinearly parameterized uncertainties are always a difficult issue to handle in adaptive control even for the single system scenario. Results on adaptive techniques to handle nonlinearly parameterized models are still rare in the existing literature. Some results were obtained, but not for a general nonlinear setting. The research based on convex/concave nonlinear functions is a major research line for adaptive control of nonlinearly parameterized models. Direct adaptive control has been investigated for systems with convexly parameterized models. It is shown in [109] that the gradient search goes to the right direction in a certain area in the state space by using the convexity condition. Non-convexly parameterized systems have been studied in [110] by using a min-max algorithm. Other results on nonconvexly parameterized systems can be found in [111, 112]. In [113], adaptive control for nonlinearly parameterized systems was studied by exploiting the monotonicity property of certain nonlinear functions. Direct and indirect adaptive algorithms were proposed in [114, 115] by identifying the monotonicity property. This method is called immersion and invariance (I&I) adaptive control. Another interesting result can be seen in [116], where an adaptive control strategy is studied based on a forward/backward update law.

The class of systems with fractional parameterization has been considered in another research line for adaptive control of nonlinearly parameterized models. In this case, the unknown constant parameters appear affinely in both numerator and denumerator. Some results have been reported for several cases. For example, an adaptive control method was proposed in [117] for fractional parameterization where uncertainties are bounded by a function with fractional parameterization. The adaptive repetitive control was developed in [118] for systems with unknown constant parameters and unknown constant time functions. Another adaptive control method for a class of strict-feedback nonlinearly parameterized systems was studied in [119] by introducing a biasing vector function into the parameter estimate.

Intelligent computation is another technique that can be applied to handle nonlinearly parameterized systems. For example, fuzzy approximators in [120] and neural networks (NN) in [121] were proposed to handle the systems with nonlinearly parameterized nonlinear dynamics. However, the intelligent system techniques like NN and fuzzy logic only approximate the nonlinear function. In adaptive computation, complex uncertain nonlinearities including linearly and nonlinearly parameterized nonlinear models are simplified using the theorem of universal approximation.

The adaptive control method was also studied for consensus of nonlinear MASs subject to uncertainties. However, there are no results available for MASs with nonlinearly parameterized dynamics. Some results have been obtained for linearly parameterized MASs. For example, consensus control for linearly parameterized MASs with an undirected graph can be found in [87] for first-order system and more general framework in [88]. A centralized adaptive consensus scheme for first-order MASs with linearly parameterized nonlinearities was developed for the leader-following case in [93] under jointly connected topologies. Further extension can be found in [94], where a pure distributed adaptive consensus was developed for leaderless MASs with linearly parameterized systems.

Intelligent computation was also studied to handle uncertainties in the nonlinear dynamics in MASs, but for the linearly parameterized model. For example, NN was proposed in [95] for first-order MASs. In [96], NN was also applied to handle linearly parameterized systems, but for second-order MASs. Further extension for higher-order MASs with linearly parameterized systems can be found in [97]. Consensus of MASs in [95, 96, 97] is achieved with a residual error. NN approximation of the nonlinear function means that a residual error exists. The residual error is also caused by distributed implementation of the adaptation law.

1.3 Main Work of the Thesis

In this thesis, our focus is on the study of distributed adaptive consensus for nonlinear MASs subject to uncertainties. The main results of this thesis are presented in Chapters 3, 4 and 5. Centralized adaptive consensus of heterogeneous nonlinear MASs with unknown constant parameters is studied in Chapter 3. Based on the results in Chapter 3 and other existing literature, we develop a distributed adaptive control scheme for consensus of heterogeneous nonlinear MASs with a linearly and nonlinearly parameterized model in Chapter 4 and 5, respectively.

In Chapter 3, we start our study on consensus of MASs with unknown constant parameters in the nonlinear dynamics using the traditional adaptive control method. The nonlinear function is assumed to be in the class of linearly parameterized nonlinear models. Each agent needs the information from the neighbours and neighbour's neighbours to update its controller. The convergence analysis is solved by using Lyapunov stability analysis and Barbalat's Lemma.

There are some limitations in the adaptive approach when applied to distributed consensus control. Based on the existing literature as presented in Subsection 1.2.2 and results in Chapter 3, we find some technical difficulties in designing a distributed adaptive control scheme to achieve asymptotic consensus for MASs with uncertain nonlinearities. From the results in Chapter 3, we find the inherent drawback in an adaptive law along the gradient of Lyapunov function to solve this open problem due to the lack of its distributed implementation. The adaptive approach in Chapter 3 can only be applied in a distributed fashion for limited cases.

In Chapter 4, we propose a novel distributed adaptive scheme, not relying on the gradient of the Lyapunov function, for general nonlinear MASs with unknown constant parameters. We introduce an input compensation as a novel idea for asymptotic consensus such that the steady state of the estimation error is not zero but a manifold in the state space of agent states and estimated parameters. Our approach is not limited to first-order MASs, but applicable to more general MASs. A distributed adaptive framework for general linearly parameterized nonlinear MASs is studied in this chapter. Then an application to our approach for second-order nonlinear MASs is presented as a case study. Information about global networked agents is not required anymore to generate the controller. Moreover, consensus can be achieved asymptotically. This means that the controller is more practical. Therefore, the inherent drawback in an adaptive law along the gradient of Lyapunov function is removed.

One of the challenges in designing adaptive control is for nonlinearly parameterized systems. The existing adaptive control methods for linearly parameterized dynamics cannot be applied anymore. Adaptive control approaches to handle this situation for an individual setting or single agent are still very rare. Some results have been obtained, however not for general nonlinear function. Moreover, none of the results have been successfully applied to MASs.

In Chapter 5, the consensus problem encountered is more complicated. The nonlinear dynamics in the MASs is nonlinearly parameterized. We extend the results in Chapter 4. A distributed adaptive framework for consensus of nonlinearly parameterized MASs is studied in this chapter. Then an application to second-order MASs is proposed as a case study.

1.4 Thesis Outline

In Chapter 1, we introduced the research background and motivation of MASs and adaptive control. Subsequently, we presented the overview of consensus of MASs with linear and nonlinear dynamics. Following that, we presented the main work and outline of the thesis.

In Chapter 2, we will review some algebraic graph theory, including basic knowledge and the associated matrices. Then we will present some preliminary knowledge about adaptive control.

In Chapter 3, we will develop consensus control for MASs subject to uncertainties using traditional adaptive control methods. In the first part, we will propose a centralized adaptive consensus framework for MASs with uncertain nonlinearities. Then an adaptive consensus control for second-order MASs will be presented as a case study. The convergence of every agent will be analyzed using Lyapunov stability and Barbalat's Lemma. We demonstrate the performance of the proposed controller using a numerical example.

In Chapter 4, we will develop a distributed consensus scheme for MASs with uncertainties. This chapter will be the extension of the control protocol in Chapter 3 and a distributed consensus control law will be designed. Before presenting our approach, we will provide problem formulation and the inherent drawback in adaptive consensus with some motivation examples. After that, we will propose a distributed adaptive framework for general linearly parameterized nonlinear MASs. Following that, we will present an application of our approach for second-order nonlinear MASs as a case study. Lyapunov stability and Barbalat's Lemma will be used to analyze consensus. To illustrate the effectiveness of our approach, a numerical example will be presented with various scenarios.

In Chapter 5, we will extend the consensus control method in Chapter 4 to more general MASs in the form of a nonlinearly parameterized model. After presenting problem formulation and preliminaries, a distributed adaptive framework will be presented for consensus of nonlinearly parameterized MASs. Following that, an application to second-order MASs will be proposed as a case study. Similar to the previous chapter, we will apply Lyapunov stability and Barbalat's Lemma to prove consensus. We will simulate the proposed control in a numerical example to demonstrate the performance of our approach.

In Chapter 6, we will summarize the whole thesis and give a short discussion for future research.

2

Preliminary Knowledge

2.1 Algebraic Graph Theory

Algebraic graph theory is a very important tool for studying MASs. The communication link among agents in a particular connected environment can be represented by a topology or graph. In the existing literature, graph theory has been extensively used to solve the consensus problem in MASs. In this section, we will present the basic concepts and some fundamental properties of graphs. More information on the graph theory for MASs can be found in [33].

2.1.1 Basic Concepts

Consider the network topology represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with n nodes (i.e., agents). Denote a finite non-empty set of nodes as $\mathcal{V} = \{1, \cdots, n\}$. The set of communication links or the set of edges is represented by $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. Denote an element of $\mathcal{E}$ as (i, j), where $i, j = 1, \cdots, n$ and $i \neq j$. An edge from i to j is also represented by an arrow, where the tail is at node i and the head is at node j. The number of edges having node i as a head is called the in-degree of i, denoted by d_i. The number of edges having node i as a tail is called the out-degree of i. We denote the adjacency matrix as $\mathcal{A} = [a_{ij}]$ and the in-degree matrix as $D = \text{diag}\{d_i\}$.

A graph is said to be a balanced graph if the in-degree and the out-degree are equal for every node i. Two examples of balanced graph can be seen in Fig. 2.1. Based on the interaction among these nodes, graphs can be divided into two categories, which are directed and undirected. In a directed graph, a parent node i receives the information from a child node j, but not vice versa. A directed graph can be said to be connected if there is a directed link from node i to j for each pair of nodes i and j. A pair of nodes i and j are strongly connected if there are two links with opposite directions between nodes i and j. Therefore, every node pair can obtain information from each other. A directed graph is said to be a connected graph if there is no isolated node in a connected topology environment. Contrary to a directed graph, in an undirected graph, two linked nodes i and j are always able to obtain information from each other, or we can say that the pairs of nodes are unordered. In an undirected graph, nodes i and j are said to be connected if there is a link between i and j, otherwise they are unconnected. An undirected topology is said to be connected if there is a link for each pair of nodes i and j. An example for undirected and directed graphs can be seen in Fig. 2.2 and 2.3 respectively.

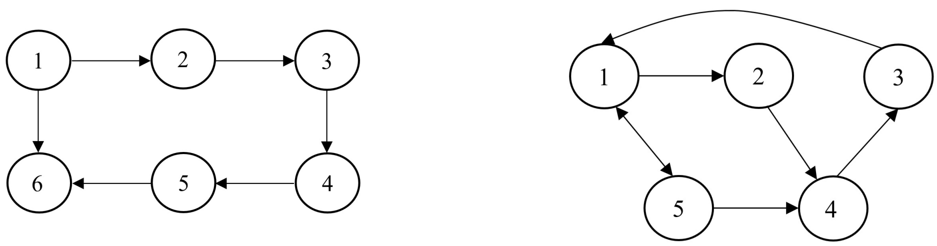

Figure 2.1: Two balanced graphs

One kind of connected directed graph is a (directed) tree, where each node has a parent, except one node, called the root. A directed graph is said to have a spanning tree if all of its nodes are in a subgraph which is a tree. A connected directed graph may have several spanning trees. At least one directed spanning tree exists in a connected

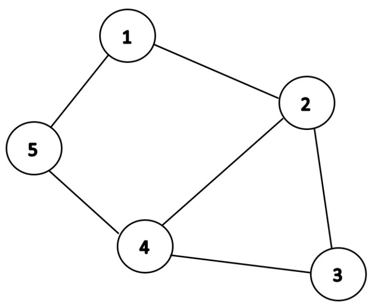

Figure 2.2: An undirected graph

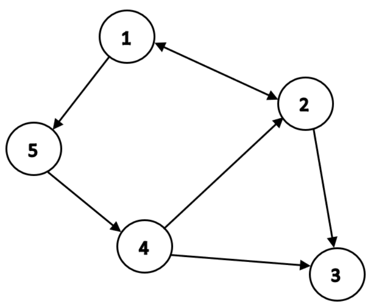

Figure 2.3: A directed graph

directed graph. As an example, a spanning tree of the directed graph in Fig. 2.3 is illustrated by bold links in Fig. 2.4.

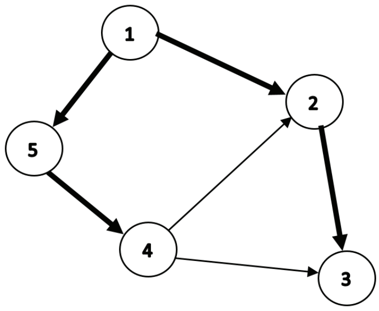

Figure 2.4: A spanning tree

2.1.2 Graph Matrices

A graph has some associated matrices commonly used in MASs. The edges weight $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$, otherwise $a_{ij} = 0$. The weighted in-degree of node i is given by

$$d_i = \sum_{j=1}^{n} a_{ij},$$

and the weighted out-degree of node i is defined as

$$d_i^o = \sum_{j=1}^{n} a_{ji}.$$

The adjacency matrix $\mathcal{A}$ is defined as $\{a_{ij}\}$ with zero diagonals. The degree matrix D is a diagonal matrix defined as $D = \text{diag}\{d_i\}$. This matrix is generated from $\mathcal{A}$ and contains information about the number of edges attached to every node. In an undirected graph, $\mathcal{A}$ is symmetric. Therefore, an undirected graph is a balanced graph.

Laplacian matrix plays a very important role in the study of MASs. The Laplacian matrix L is generated using matrices $\mathcal{A}$ and D such that $L = D - \mathcal{A} = [L_{ij}] \in \mathbb{R}^{n \times n}$. In an undirected graph, L is a symmetric matrix. In a connected graph, L is a positive semi-definite matrix that at least has one zero eigenvalue and the other eigenvalues have positive real parts. Hence L has rank $n - 1$. Note that the sum of every row of L is equal to zero.

The following is an example to generate $\mathcal{A}, D$ and L. Consider a directed graph in Fig. 2.3. Let every edge weight be 1. Then $\mathcal{A}$ and D are

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \; D = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

hence the Laplacian matrix L is

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

2.2 Adaptive Control

This section contains a brief introduction to adaptive control. Consider a first-order system with uncertain nonlinear dynamics

$$\dot{\eta} = \vartheta\eta + u, \tag{2.1}$$

where η is the state, u is control input and ϑ is an unknown constant parameter.

Noting that if ϑ were known for feedback control design, by selecting

$$u = -\kappa_1\eta - \vartheta\eta,$$

where $\kappa_1 > 0$, then the equilibrium point of system (2.1) is globally asymptotically stable.

However, ϑ is unknown in many practical situations. There are two common approaches that propose to handle system dynamics with uncertainties, specially with unknown constant parameters. The first approach is a fixed gain feedback control. Under this scheme, a fixed gain feedback control is designed to handle the changing parameters within the bound of uncertainties. The idea behind a fixed-gain control is to tune the gain to dominate the uncertain dynamics. For system (2.1), the stability is guaranteed by selecting $u = -\kappa\eta$ as long as $\kappa > |\vartheta|$ is satisfied. The weakness of this approach is that the bound must be known in advance.

The second approach is adaptive control. This is a systematic control technique with the capability to adapt itself to handle the changing environment, such as unknown constant parameters. An adaptive controller can be designed without *a priori* information about the bound of uncertainties. In this scheme, the adaptation law will estimate the unknown parameters in real time, hence the controller is adjustable. Compared with a fixed gain feedback control, adaptive control has an additional block, i.e., an adaptation law that allows the controller to update itself. Hence the damping in the system response can be handled when the changing parameters occur. The idea behind adaptive control is conceptually simple. The parameter estimate is generated in the block of adaptation law, then it becomes additional information for the controller to improve itself as if it were the true parameter. The estimate will be continuously updated until convergence with the true value of the unknown parameters. The concept of this scheme is called the certainty equivalence principle. More detailed information about the adaptive approach can be found in [100, 122].

Now, we design an adaptive controller for the system (2.1). We select the corresponding certainty-equivalent controller

$$u = -\kappa_1 \eta - \kappa_2 \hat{\vartheta} \eta,$$

where $\kappa_1, \kappa_2 > 0$ and $\hat{\vartheta}$ is the estimate of ϑ generated by adaptive law. We choose the Lyapunov function of system (2.1) to be

$$U(\eta, \hat{\vartheta}) = \frac{1}{2}\eta^2 + \frac{1}{2\kappa_2}(\hat{\vartheta} - \vartheta)^2.$$

By selecting

$$\dot{\hat{\vartheta}} = \kappa_2 \eta^2,$$

The time-derivative of $U(\eta, \hat{\vartheta})$ along the closed-loop system is

$$\begin{aligned} \dot{U}(\eta, \hat{\vartheta}) &= \eta\dot{\eta} + \frac{1}{\kappa_2}\dot{\hat{\vartheta}}(\hat{\vartheta} - \vartheta) \\ &= -\kappa_1\eta^2 - \eta^2(\hat{\vartheta} - \vartheta) + \frac{1}{\kappa_2}\dot{\hat{\vartheta}}(\hat{\vartheta} - \vartheta) \\ &= -\kappa_1\eta^2. \end{aligned} \tag{2.2}$$

It is clear to see that both η and $\hat{\vartheta} - \vartheta$ are bounded. By Barbalat's Lemma, then $\lim_{t\to\infty} \eta = 0$.

3

Centralized Adaptive Consensus of Multi-Agent Systems

3.1 Introduction

Research on cooperative control of MAS has been growing progressively in recent years. Consensus is an important topic in cooperative control of MASs. The basic idea of a consensus approach is to drive the state or output of every agent to a common value (agreement). Many results have been obtained for MASs with linear dynamics as we discussed in the literature review section in Chapter 1. We can conclude that research on first and second-order MASs with linear dynamics is relatively mature in the existing literature.

Research on MASs with nonlinear dynamics is a more challenging problem. At the beginning, consensus problems were studied for nonlinear MASs satisfying Lipschitz conditions. In some cases, linear consensus controllers are still able to handle this. However, in many practical situations, the nonlinear dynamics of MASs may not satisfy Lipschitz conditions, hence linear consensus controllers cannot be applied anymore. Consequently, a nonlinear controller is required. In practice, the system dynamics may contain uncertain nonlinearities that may differ for different agents. This induces heterogeneity in the MASs and makes the consensus problem more complicated.

In this chapter, we study a centralized adaptive consensus framework for nonlinear heterogeneous MASs subject to uncertainties. A control protocol is designed to achieve consensus by maintaining the collective nominal behaviour for all agents.

We organize the remainder of this chapter as follows. The problem formulation is presented in Section 3.2. In Section 3.3, we present a centralized adaptive consensus framework for MASs to maintain its nominal dynamics behaviour with the presence of uncertainties in the nonlinear dynamics. An application to second-order MASs under an undirected network is presented in Section 3.4. In Section 3.5, a numerical example is given to illustrate the effectiveness of our approach. We close this chapter with a summary in the last section.

3.2 Problem Formulation

Consider a MAS with $n \geq 2$ autonomous agents expressed by

$$\dot{x}_i = f_i(x), \; i = 1, \cdots, n \tag{3.1}$$

where $x_i \in \mathbb{R}^l$ is the state of the i and $f_i(x)$ is a general function representing the agent dynamics. System dynamics (3.1) has a compact form

$$\dot{x} = f(x), \tag{3.2}$$

where

$$\begin{aligned} x &= [x_1^\mathsf{T}, x_2^\mathsf{T}, \cdots, x_n^\mathsf{T}]^\mathsf{T} \\ f(x) &= [f_i^\mathsf{T}(x_1), f_2^\mathsf{T}(x_2), \cdots, f_n^\mathsf{T}(x_n)]^\mathsf{T}. \end{aligned}$$

This is a collective nominal behaviour of closed-loop MAS without the presence of uncertainties in the dynamics. MAS (3.2) is supposed to achieve consensus with a particular collective dynamics behaviour. This is described by a property in terms of a Lyapunov-like function $V(x)$ satisfying the assumption below.

Assumption 3.2.1 There exists a continuously differentiable function $V(x)$ satisfying

$$\underline{\alpha}(\|x\|_R) \leq V(x) \leq \bar{\alpha}(\|x\|_R)^1$$

for a matrix $R \in \mathbb{R}^{\bar{n}l \times nl}$ with $\bar{n} \leq n$ and class $\mathcal{K}_\infty$ functions $\underline{\alpha}$ and $\bar{\alpha}$, such that,

$$\frac{\partial V(x)}{\partial x} f(x) \leq -\alpha(\|x\|_R) \tag{3.3}$$

for a class $\mathcal{K}_\infty$ function α. ∎

[1] Throughout the thesis, the notation $\|x\|_R^2 = x^\mathsf{T} R^\mathsf{T} R x$ is used.

Remark 3.2.1 For $x_i \in \mathbb{R}$, if a full row matrix $R \in \mathbb{R}^{(n-1)l \times nl}$ is perpendicular to $\mathbf{1} = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}^\top \in \mathbb{R}^n$, then $\|x\|_R = 0$ implies $x = x_o\mathbf{1}$ for some $x_o \in \mathbb{R}$, that is a typical consensus phenomenon. ∎

Now, we consider the presence of uncertainties in the MAS. The objective is to propose an adaptive consensus protocol to handle the uncertainties in the dynamics such that consensus is achieved by maintaining the collective behaviour of the nominal system. The adaptive law will be designed separately from the consensus protocols in the nominal dynamics system. This situation is implicitly shown in the closed-loop systems (4.1).

Consider the MAS subject to uncertainties with $n \geq 2$ autonomous agents described by

$$\dot{x}_i = f_i(x) + g_i(x_i, w_i, \mu_i),\ i = 1, \cdots, n \tag{3.4}$$

where the nonlinear function $g_i(x_i, w_i, \mu_i) \in \mathbb{R}^l$ contains constant unknown parameters w_i and an additional control input μ_i to adaptively handle the uncertainties. Suppose the uncertainties to be in the class of linearly parameterized structure, i.e.,

$$g_i(x_i, w_i, \mu_i) = h_i(x_i)(w_i - \mu_i). \tag{3.5}$$

We can rewrite the closed-loop system of MAS (3.4) in a compact form as follows

$$\dot{x} = f(x) + H(x)(w - \mu), \tag{3.6}$$

where

$$\begin{aligned} w &= [w_1^\top, w_2^\top, \cdots, w_n^\top]^\top, \\ \mu &= [\mu_1^\top, \mu_2^\top, \cdots, \mu_n^\top]^\top, \\ H(x) &= \operatorname{diag}\begin{bmatrix} h_1(x_1) & h_2(x_2) & \cdots & h_n(x_n) \end{bmatrix}. \end{aligned}$$

The uncertain nonlinear functions $g_i(x_i, w_i, \mu_i)$ could be trivially cancelled by $\mu_i = w_i$ if the parameter w_i were known. In the practical situation, an unknown w_i may exist in the dynamics, hence an additional controller is required to handle the unknown constant parameters.

3.3 A Centralized Adaptive Scheme

In this chapter, an adaptive controller is designed along the gradient of the Lyapunov function $V(x)$ for the case with unknown w_i. The centralized adaptive scheme is summarized as follows.

Theorem 3.3.1 *For the system (3.4) with (3.5) under Assumption 3.2.1, with the controller*

$$\begin{aligned} \mu &= \hat{w} \\ \dot{\hat{w}}^{\mathsf{T}} &= \lambda \frac{\partial V(x)}{\partial x} H(x), \ \lambda > 0 \end{aligned} \tag{3.7}$$

the time-derivative of

$$U(x, \tilde{w}) = V(x) + \frac{1}{2\lambda} \tilde{w}^{\mathsf{T}} \tilde{w} \tag{3.8}$$

with $\tilde{w} = \hat{w} - w$ satisfies

$$\dot{U}(x, \tilde{w}) \leq -\alpha(\|x\|_R), \tag{3.9}$$

along the trajectory of the closed-loop system (3.4)+(3.5)+(3.7). ∎

Proof: Direct calculation shows that the derivative of $V(x)$ along the dynamics (3.4) with (3.5) satisfies

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V(x)}{\partial x} f(x) - \frac{\partial V(x)}{\partial x} H(x) \tilde{w} \\ &\leq -\alpha(\|x\|_R) - \frac{\partial V(x)}{\partial x} H(x) \tilde{w}. \end{aligned}$$

Hence,

$$\begin{aligned} \dot{U}(x, \tilde{w}) &\leq -\alpha(\|x\|_R) - \frac{\partial V(x)}{\partial x} H(x) \tilde{w} + \frac{1}{\lambda} \dot{\tilde{w}}^{\mathsf{T}} \tilde{w} \\ &\leq -\alpha(\|x\|_R) \end{aligned}$$

for $\dot{\tilde{w}} = \dot{\hat{w}}$ given in (3.7). ■

The adaptive law (3.7) can be rewritten as follows, for $i = 1, \cdots, n$,

$$\begin{aligned} \mu_i &= \hat{w}_i \\ \dot{\hat{w}}_i^{\mathsf{T}} &= \lambda \frac{\partial V(x)}{\partial x_i} h_i(x_i), \ \lambda > 0. \end{aligned} \tag{3.10}$$

From (3.10), we can see that both the local state of agent i and full network state x are required in designing the controller. The Lyapunov function $V(x)$ for the nominal behaviour system is required to be designed in a centralized manner.

Remark 3.3.1 The adaptive control (3.7) in Theorem 3.3.1 is designed in a centralized fashion. A collective nominal dynamics behaviour $f_i(x)$ is typically implemented in a distributed fashion beforehand by assuming that there are no uncertainties in the dynamics. In the next section, application to Theorem 3.3.1 will be implemented for second-order MASs with unknown constant parameters. ∎

3.4 Application to A Network of Second-Order Uncertain Dynamics

Consider a group of $n \geq 2$ autonomous agents described by the set of equations

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \xi_i\left(w_i, v_i\right) + u_i, \; i = 1, \ldots, n, \end{aligned} \tag{3.11}$$

where $p_i, v_i, u_i \in \mathbb{R}$ are the position, velocity and control input of the agent i respectively. The nonlinear function $\xi_i(w_i, v_i) = \zeta_i(v_i) w_i$ represents heterogeneous nonlinearities with known function $\zeta_i(v_i)$ and an unknown constant parameter w_i.

For more convenience of presentation, we can rewrite MAS (3.11) as

$$\dot{x}_i = A x_i + h_i(x_i, w_i) + u_i, \tag{3.12}$$

and in a compact form as

$$\dot{x} = Ax + H(x, w) + u \tag{3.13}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; x_i = \begin{bmatrix} p_i \\ v_i \end{bmatrix}, \; h_i(x_i, w_i) = \begin{bmatrix} 0 \\ \zeta_i(v_i) w_i \end{bmatrix}, \\ p &= \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, \; v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \; x = \begin{bmatrix} p \\ v \end{bmatrix}, \\ \zeta(v) &= \operatorname{diag}\begin{bmatrix} \zeta_1(v_1) & \vdots & \zeta_n(v_n) \end{bmatrix}, \; w = \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}, \\ H(x, w) &= \begin{bmatrix} \mathbf{0}_n \\ \zeta(v) w \end{bmatrix}, \; u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}. \end{aligned}$$

In this section, the network topology is represented by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \cdots, n\}$ is a finite non-empty set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of undirected edges. Let us denote the adjacency matrix as $\mathcal{A} = [a_{ij}]$, where the edge from i to j is denoted as $a_{ij} > 0$ if there exists a communication link from i to j. The in-degree matrix of undirected network is represented by $D = \operatorname{diag}(d_i)$. The Laplacian matrix is defined as $L = D - \mathcal{A} = [L_{ij}] \in \mathbb{R}^{n\times n}$, which has elements of $L_{ij} = -a_{ij}$ for $j \neq i$ and $L_{ii} = \sum_{j=1, j\neq 1}^{n} a_{ij}$.

Assumption 3.4.1 The network is connected. ∎

Consider the virtual reference for the networked agents (3.11) is described as

$$\begin{aligned} \dot{p}_o &= v_o \\ \dot{v}_o &= 0. \end{aligned} \tag{3.14}$$

There are two typical scenarios for MAS to achieve consensus by following the agreed trajectory (3.14):

(i) Assigned reference trajectory

In this case, the reference trajectory (3.14) is prescribed as *a priori* information. Or, in other words, the relative position and velocity between agent i and the leader are always available for feedback control design, then the consensus problem simply reduces to distributed control for each agent. This is a type of traditional control problem. However, if the reference trajectory is available only to some agents or one agent, then consensus becomes more complicated. An interesting result for linear MASs can be seen in [123], and results for nonlinear MASs in [96].

(ii) Unassigned reference trajectory

In this setting, the reference trajectory (3.14) is not prescribed *a priori*. It means that the relative position and velocity between agent i and the leader are not available for feedback control design for any agent. The consensus problem in this situation is more complicated. Consensus of MAS with nonlinear dynamics under this setting is limited in the literature. Some results can be found in [79, 81, 82] by applying robust control approaches.

In this application, we investigate the setting of a general undirected leaderless MAS with uncertain nonlinearities under Assumption 3.4.1, where the reference trajectory (3.14) is not prescribed *a priori*. The objective of asymptotic consensus is

$$
\begin{aligned}
\lim_{t\to\infty} p_i(t) - p_o(t) &= 0 \\
\lim_{t\to\infty} v_i(t) - v_o(t) &= 0
\end{aligned}
\tag{3.15}
$$

for some time functions $p_o(t), v_o(t) : [0,\infty) \mapsto \mathbb{R}$.

Recall that the Laplacian matrix L has at least one zero eigenvalue and the rest of the eigenvalues have positive real parts. It has the following property

$$
L\mathbf{1} = \mathbf{1}^{\intercal} L = 0,
$$

where $\mathbf{1} = \left[\begin{array}{ccc} 1 & \cdots & 1 \end{array}\right]^{\intercal}$. There exists a matrix $U_1 \in \mathbb{R}^{(n)\times n-1}$ such that

$$
\bar{U} = \left[\begin{array}{cc} \frac{1}{\sqrt{n}}\mathbf{1} & U_1 \end{array}\right]
$$

is an orthogonal matrix. As a result, the Laplacian matrix L can be transformed into

$$
\bar{U}^{-1} L \bar{U} = \left[\begin{array}{cc} 0 & 0 \\ 0 & \bar{H} \end{array}\right],
$$

where $\bar{H}$ is a positive definite matrix with all positive eigenvalues of L on the diagonal. Let us define the matrix R as follows

$$
\left[\begin{array}{c} U_1^{\intercal} p \\ U_1^{\intercal} v \end{array}\right] = Rx, \tag{3.16}
$$

where R has a full rank and the rows of R are perpendicular to span $\{\mathbf{1} \otimes I_2\}$. From the definition of $\bar{U}$ and $\bar{U}^{-1}$, one has

$$
\frac{1}{n}\mathbf{1}\mathbf{1}^{\intercal} + U_1 U_1^{\intercal} = I \tag{3.17}
$$

and

$$
\begin{aligned}
\bar{H} U_1^{\intercal} &= U_1^{\intercal} L U_1 U_1^{\intercal} \\
&= U_1^{\intercal} L \left(I - \frac{1}{n}\mathbf{1}\mathbf{1}^{\intercal} \right) \\
&= U_1^{\intercal} L
\end{aligned}
$$

Before giving the proof for the control protocol for MAS (3.11), let us introduce two technical lemmas which will be used to prove Theorem 3.4.1.

Lemma 3.4.1 *[33] The network is under Assumption 3.4.1. Let $\gamma_1, \gamma_2 \in \mathbb{R}$ be the weights of the consensus protocol and $\Lambda_i = 2, 3, \ldots, n$ is the positive eigenvalue of L. The matrix*

$$\bar{A} = \begin{bmatrix} 0 & I_{n-1} \\ -\gamma_1 \bar{H} & -\gamma_2 \bar{H} \end{bmatrix}$$

has stable eigenvalues if and only if the consensus weights satisfy $\gamma_1, \gamma_2 > 0$ and they are chosen as follows:

(i) If the i-th eigenvalue Λ_i of L is positive and real, then $\gamma_1, \gamma_2 > 0$.

(ii) If Λ_i is complex, then

$$\gamma_1 \gamma_2 > max_i \frac{\mathrm{Im}^2\{\Lambda_\mathrm{i}\} - \mathrm{Re}^2\{\Lambda_\mathrm{i}\}}{\mathrm{Re}^2\{\Lambda_\mathrm{i}\} \left|\Lambda_\mathrm{i}\right|^2}.$$

∎

Lemma 3.4.2 *The network is under Assumption 3.4.1, consider MAS (3.11) with $\xi_i(w_i, v_i) = 0$. Consensus is achieved by taking the following control protocol*

$$u = -\gamma_1 L p - \gamma_2 L v, \tag{3.18}$$

where γ_1 and γ_2 are properly selected such that the matrix $\bar{A}$ is Hurwitz. Let $P = \begin{bmatrix} \bar{P} & \bar{H} \\ \bar{H} & \bar{H} \end{bmatrix} > 0 \in \mathbb{R}^{(2n-2)\times(2n-2)}$ is a unique solution to the Lyapunov equation

$$P\bar{A} + \bar{A}^\intercal P = -Q < 0,$$

where $\bar{P} > 0 \in \mathbb{R}^{(n-1)\times(n-1)}$. The time-derivatives of Lyapunov function $V(x)$

$$V(x) = x^\intercal R^\intercal P R x \tag{3.19}$$

satisfying

$$\lambda_{\min}(P)\|x\|_R^2 \leq V(x) \leq \lambda_{\max}(P)\|x\|_R^2$$

along the closed-loop system is

$$\dot{V}(x) \leq -\lambda_{\min}(Q)\|x\|_R^2. \tag{3.20}$$

∎

Proof: We can write the closed-loop system of MAS (4.15) with free uncertainties and under controller (3.18) as

$$\begin{aligned}\dot{p} &= v \\ \dot{v} &= -\gamma_1 Lp - \gamma_2 Lv. \end{aligned} \tag{3.21}$$

By doing the calculation using the facts in T and T^{-1}, we have

$$\begin{aligned} R\dot{x} &= \left[\begin{array}{c} U_1^{\mathsf{T}}\dot{p} \\ U_1^{\mathsf{T}}\dot{v} \end{array}\right] \\ &= \left[\begin{array}{c} U_1^{\mathsf{T}}v \\ -\gamma_1 U_1^{\mathsf{T}}Lp - \gamma_2 U_1^{\mathsf{T}}Lv \end{array}\right] \qquad (3.22) \\ &= \left[\begin{array}{c} U_1^{\mathsf{T}}v \\ -\gamma_1 \bar{H} U_1^{\mathsf{T}}p - \gamma_2 \bar{H} U_1^{\mathsf{T}}v \end{array}\right] \\ &= \left[\begin{array}{cc} 0 & I_{n-1} \\ -\gamma_1 \bar{H} & -\gamma_2 \bar{H} \end{array}\right] \left[\begin{array}{c} U_1^{\mathsf{T}}p \\ U_1^{\mathsf{T}}v \end{array}\right] \\ &= \bar{A}Rx. \qquad (3.23) \end{aligned}$$

Now, we can calculate the time-derivatives of Lyapunov function (3.19)

$$\begin{aligned} \dot{V}(x) &= x^{\mathsf{T}}R^{\mathsf{T}}PR\dot{x} + \dot{x}^{\mathsf{T}}R^{\mathsf{T}}PRx \\ &= x^{\mathsf{T}}R^{\mathsf{T}}P\bar{A}Rx + [\bar{A}Rx]^{\mathsf{T}}PRx \\ &= x^{\mathsf{T}}R^{\mathsf{T}}(P\bar{A} + \bar{A}^{\mathsf{T}}P)Rx \\ &= x^{\mathsf{T}}R^{\mathsf{T}}QRx \\ &\leq -\lambda_{\min}(Q)\|x\|_R^2. \end{aligned} \tag{3.24}$$

The proof is completed. ∎

The main result on a centralized adaptive controller for MAS (3.11) is stated in the Theorem 3.4.1.

Theorem 3.4.1 *The graph is under Assumption 3.4.1. Consider MAS (3.11), consensus is achieved in the sense of (3.15) for some time functions $p_o(t), v_o(t) : [0, \infty) \mapsto \mathbb{R}$ by taking the following control protocol*

$$u = -\gamma_1 Lp - \gamma_2 Lv - \zeta(v)\mu \tag{3.25}$$

where the consensus weight γ_1 *and* γ_2 *are provided in Lemma 3.4.2 and*

$$\begin{aligned} \mu &= \hat{w} \\ \dot{\hat{w}} &= \gamma\zeta(v)L(p+v), \end{aligned} \tag{3.26}$$

with a positive constant γ. ∎

Proof: The closed-loop system of MAS (3.13) with controller (3.25) can be written as

$$\dot{x} = \bar{A}x - H(x, \tilde{w}), \tag{3.27}$$

where

$$H(x, \tilde{w}) = \begin{bmatrix} \mathbf{0}_n \\ \zeta(v)\tilde{w} \end{bmatrix}.$$

From Lemma 3.4.2, we can see that $\dot{x} = f(x)$ in this application is represented by MAS in ideal situation 3.21.

The Lyapunov function of MAS (3.27) is chosen to be

$$U(x, \tilde{w}) = V(x) + \frac{1}{\gamma}\tilde{w}^{\mathsf{T}}\tilde{w}.$$

The time-derivatives of $U(x, \tilde{w})$ along the closed-loop system is

$$\begin{aligned} \dot{U}(x, \tilde{w}) &= x^{\mathsf{T}}R^{\mathsf{T}}PR\dot{x} + \dot{x}^{\mathsf{T}}R^{\mathsf{T}}PRx + \frac{1}{\gamma}\dot{\tilde{w}}^{\mathsf{T}}\tilde{w} + \frac{1}{\gamma}\tilde{w}^{\mathsf{T}}\dot{\tilde{w}} \\ &= x^{\mathsf{T}}R^{\mathsf{T}}(P\bar{A} + \bar{A}^{\mathsf{T}}P)Rx - x^{\mathsf{T}}R^{\mathsf{T}}PRH(x, \tilde{w}) - H^{\mathsf{T}}(x, \tilde{w})R^{\mathsf{T}}PRx \\ &\quad + (p^{\mathsf{T}} + v^{\mathsf{T}})L\zeta(v)\tilde{w} + \tilde{w}^{\mathsf{T}}\zeta(v)L(p+v) \\ &= -x^{\mathsf{T}}R^{\mathsf{T}}QRx - (p^{\mathsf{T}} + v^{\mathsf{T}})U_1\bar{H}U_1^{\mathsf{T}}\zeta(v)\tilde{w} - \tilde{w}^{\mathsf{T}}\zeta(v)U_1\bar{H}U_1^{\mathsf{T}}(p+v) \\ &\quad + (p^{\mathsf{T}} + v^{\mathsf{T}})L\zeta(v)\tilde{w} + \tilde{w}^{\mathsf{T}}\zeta(v)L(p+v) \\ &= -x^{\mathsf{T}}R^{\mathsf{T}}QRx - (p^{\mathsf{T}} + v^{\mathsf{T}})L\zeta(v)\tilde{w} - \tilde{w}^{\mathsf{T}}\zeta(v)L(p+v) \\ &\quad + (p^{\mathsf{T}} + v^{\mathsf{T}})L\zeta(v)\tilde{w} + \tilde{w}^{\mathsf{T}}\zeta(v)L(p+v) \\ &\leq -\lambda_{\min}(Q)\|x\|_R^2 \end{aligned} \tag{3.28}$$

We can see that both $\|x(t)\|_R$ and $z(t)$ are bounded. By Barbalat's Lemma, one has $\lim_{t\to\infty} \|x(t)\|_R = 0$, that is,

$$\lim_{t\to\infty} \begin{bmatrix} U_1^{\mathsf{T}}p(t) \\ U_1^{\mathsf{T}}v(t) \end{bmatrix} = 0. \tag{3.29}$$

Let $p_o(t) = \frac{1}{n}\mathbf{1}^\mathsf{T} p(t)$ and $v_o(t) = \frac{1}{n}\mathbf{1}^\mathsf{T} v(t)$. From the following relationship

$$\begin{aligned} p &= \bar{U}\bar{U}^{-1}p \\ &= \begin{bmatrix} \frac{1}{\sqrt{n}}\mathbf{1} & U_1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{n}}\mathbf{1}^\mathsf{T} p \\ U_1^\mathsf{T} p \end{bmatrix} \\ &= \mathbf{1}(\frac{1}{n}\mathbf{1}^\mathsf{T} p) + U_1(U_1^\mathsf{T} p), \end{aligned}$$

and

$$\begin{aligned} v &= \bar{U}\bar{U}^{-1}v \\ &= \begin{bmatrix} \frac{1}{\sqrt{n}}\mathbf{1} & U_1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{n}}\mathbf{1}^\mathsf{T} v \\ U_1^\mathsf{T} v \end{bmatrix} \\ &= \mathbf{1}(\frac{1}{n}\mathbf{1}^\mathsf{T} v) + U_1(U_1^\mathsf{T} v), \end{aligned}$$

one has

$$\begin{aligned} \lim_{t\to\infty} p(t) - \mathbf{1}p_o(t) &= U_1 \lim_{t\to\infty} U_1^\mathsf{T} p(t) = 0 \\ \lim_{t\to\infty} v(t) - \mathbf{1}v_o(t) &= U_1 \lim_{t\to\infty} U_1^\mathsf{T} v(t) = 0. \end{aligned}$$

This completes the proof. ■

Remark 3.4.1 The control protocol (3.25) is composed of two parts. The first part is the controller (3.18) proposed to achieve consensus for the ideal situation, where the nonlinear function $\xi_i(w_i, v_i)$ vanishes. The second part is the adaptive controller (3.26) added to the controller when the uncertain nonlinear dynamics $\xi_i(w_i, v_i)$ is taken into account. These two parts can be designed separately as stated in the Theorem 3.4.1. ▮

Remark 3.4.2 Although adaptive law (3.7) is designed along the gradient of Lyapunov function in a centralized manner as stated in the Theorem 3.3.1, it can, however, be applied in a distributed fashion as a special case. MAS (3.13) with controller (3.25) and adaptive law (3.26) is one of the examples. Let

$$\begin{aligned} L_i p &= -\sum_{j=1}^{n} a_{ij}(p_j - p_i) \\ L_i v &= -\sum_{j=1}^{n} a_{ij}(v_j - v_i), \end{aligned}$$

where L_i is the i-th row of L. Then (3.13) and (3.26) can be rewritten in distributed algorithm as follows

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v - \zeta_i(v_i)\mu,$$

$$\begin{aligned}\mu_i &= \hat{w}_i \\ \dot{\hat{w}}_i &= \gamma \zeta_i(v_i) L_i(p+v).\end{aligned}$$

∎

3.5 Numerical Simulation

Consider a MAS with $n = 6$ autonomous agents described by

$$\begin{aligned}\dot{p}_i &= v_i \\ \dot{v}_i &= \zeta_i(v_i)w_i + u_i,\ i = 1, \ldots, n, .\end{aligned} \tag{3.30}$$

The function $\zeta_i(v_i)$'s are given as follows

$$\zeta_i(v_i) = \begin{cases} \sin v_i^3, & i = 1,2 \\ \cos v_i, & i = 3,4 \\ \sin v_i^2, & i = 5,6 \end{cases}.$$

Assume all the unknown parameters are selected within the interval $[-20, 20]$. The network topology is given in Fig. 3.1.

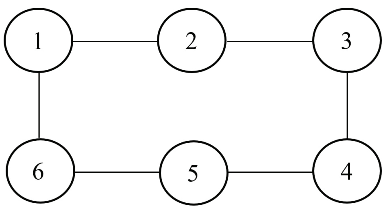

Figure 3.1: An undirected network topology of six agents

From Fig. 3.1, we can generate the Laplacian matrix

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}.$$

In this section, we will present several simulation results with various scenarios to see the effectiveness of the proposed controller. The initial conditions are chosen as follows

$$\begin{aligned} p(0) &= \begin{bmatrix} 2 & 3 & 2.5 & 0 & -3 & -0.5 \end{bmatrix}^{\mathsf{T}} \\ v(0) &= \begin{bmatrix} 3 & 2 & -3 & 2.5 & -1 & 1.5 \end{bmatrix}^{\mathsf{T}}, \end{aligned}$$

and the unknown constant parameters are

$$w = \begin{bmatrix} 20 & -10 & 6 & -4 & -14 & -20 \end{bmatrix}^{\mathsf{T}},$$

In the first scenario, we demonstrate the simulation of MAS (3.30) with $u_i = 0$. Fig. 3.2 shows that consensus is not achieved under this situation. Each agent moves without controller from its initial position and velocity according to its own nominal behaviour and nonlinear dynamics.

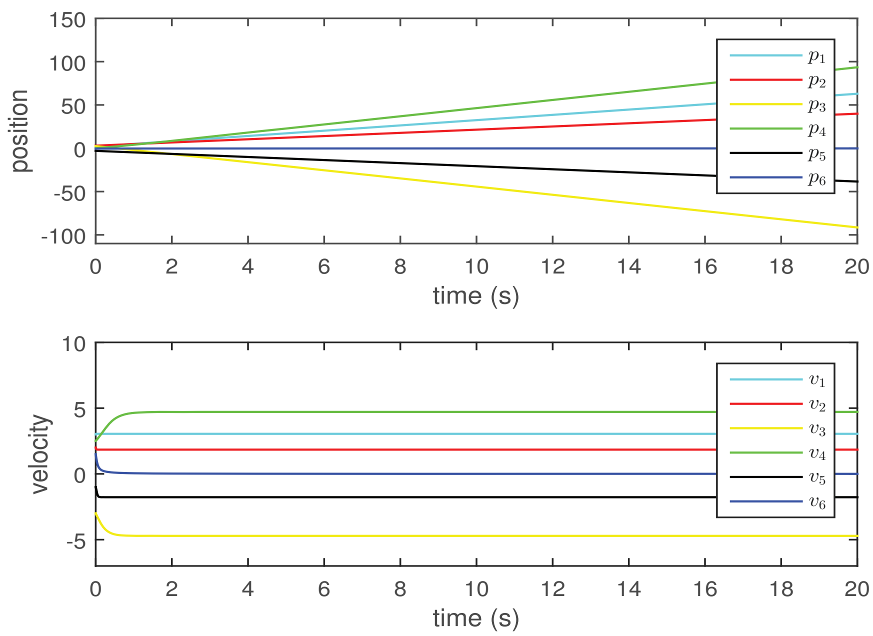

Figure 3.2: Profile of state trajectories of six agents without controller

Now, we consider MAS (3.30) when free of uncertainties i.e. $\zeta_i(v_i)w_i = 0$. The consensus controller studied in Lemma 3.4.2 is implemented for each agent. By Lemma 3.4.1, for $\bar{P} = 2I_{n-1}$, we select $\gamma_1 = 5$ and $\gamma_2 = 5$, then $\bar{A}$ is Hurwitz and Q is positive definite. Under this situation, consensus is guaranteed to be achieved under controller (3.18). Or, in other words, this situation represents MAS under ideal condition $\dot{x} = f(x)$ as described in (3.2). The simulation results for this setting can be seen in Fig. 3.3. In the plotted figure, we show that consensus is achieved as concluded in Lemma 3.4.2.

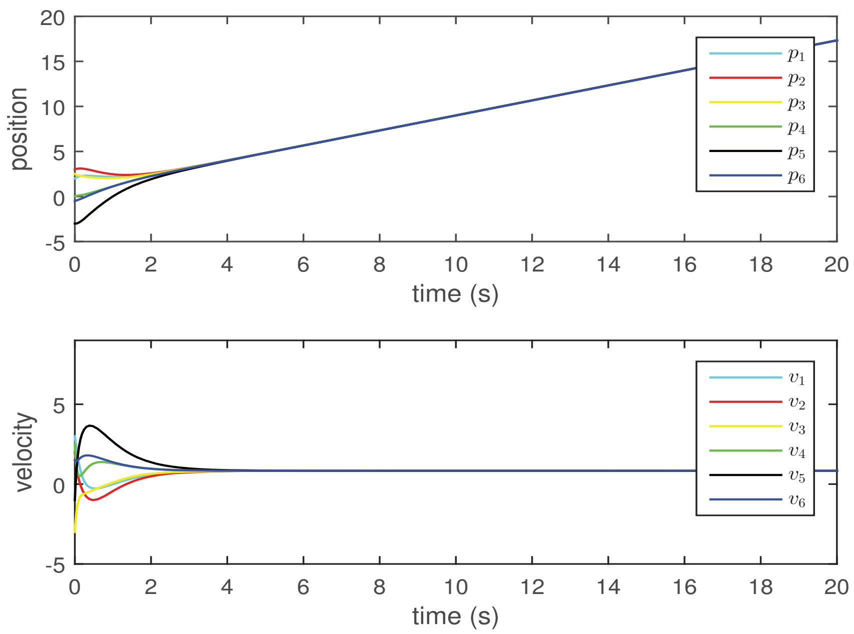

Figure 3.3: Profile of state trajectories of six agents under ideal situation

When the nonlinear dynamics $\zeta_i(v_i)w_i$ is taken into account, then an adaptive controller is required to be added in the control structure to handle the uncertain nonlinearities. By Theorem 3.4.1, the adaptive controller is generated by the following adaptation law

$$\dot{\hat{w}} = \gamma\zeta(v)L(p+v),$$

where $\gamma = 100$.

To see the performance and the effectiveness of consensus control studied in Theorem 3.4.1, we compare the response of the closed-loop system with and without adaptive controller. Fig. 3.4 and 3.5 illustrate the profile of state trajectories of six agents to achieve consensus without and with adaptive controller respectively. From the simulation results, we can see that consensus cannot be achieved for MAS with linear consensus protocol only. The controller (3.18) is not enough to guarantee consensus due to the presence of disturbance in the dynamics. By adding adaptive controller, the uncertain nonlinearities can be handled. Fig. 3.6 shows that the estimator $\hat{w}_i$ converges to the true value of unknown constant parameters as well as the dynamics of adaptive law $\dot{\hat{w}}_i$ converging to zero. The estimation error $\tilde{w}_i$ also converges to zero as can be seen in Fig. 3.7. Therefore, consensus can be achieved as concluded in Theorem 3.4.1.

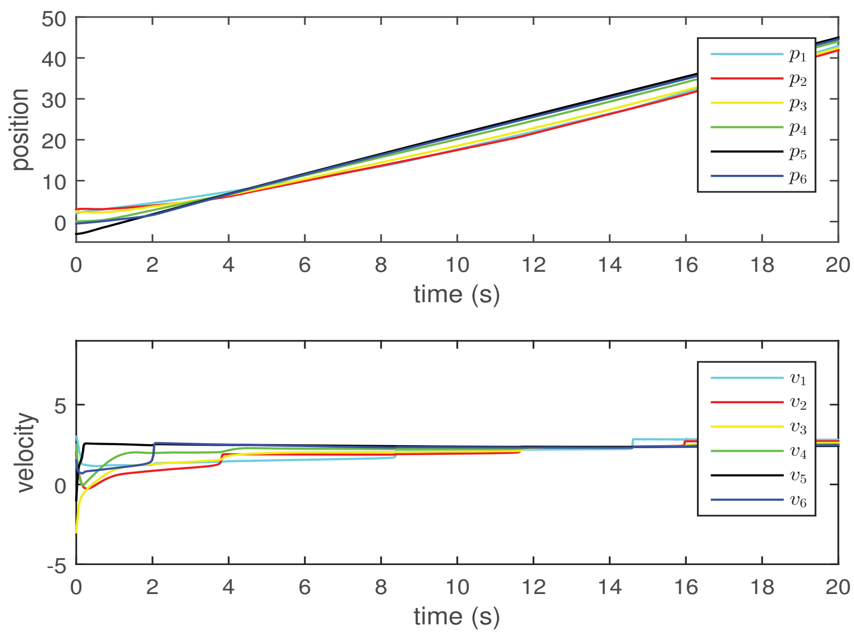

Figure 3.4: Profile of state trajectories of six agents without adaptive controller

When the range of unknown constant parameters interval is increased to be $[-100, 100]$, under similar controller, consensus still can be achieved. In this scenario, w_i is selected to be

$$w_i = \begin{bmatrix} 100 & 80 & 30 & -20 & -70 & -100 \end{bmatrix}^\top.$$

The profile of state trajectories of six agents is illustrated in Fig. 3.8. Compared with the case where unknown constant parameters are within the interval $[-20, 20]$, the controller now requires a little bit more time to achieve consensus. From Fig. 3.9 and 3.10, we can see that both $\dot{\hat{w}}_i$ and $\tilde{w}_i$ converge to zero. This means that the adaptive controller is able to estimate the unknown constant parameters. This situation shows that the adaptive controller has the capability to cope with the changing environment.

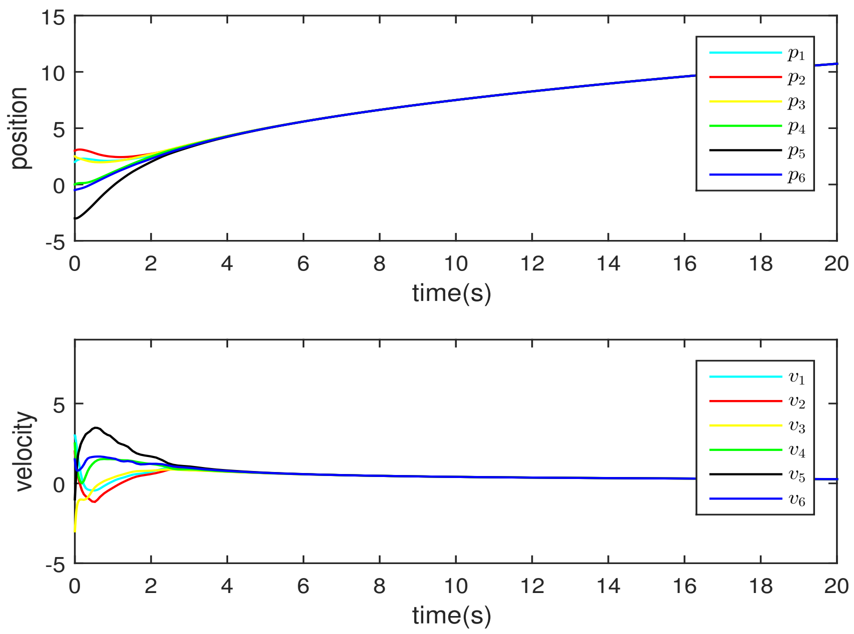

Figure 3.5: Profile of state trajectories of six agents with adaptive controller

Based on the aforementioned simulation results, we can verify that asymptotic consensus can be achieved using the proposed controller as concluded in Theorem 3.4.1. We also can see the performance of the adaptive controller to handle uncertain nonlinearities.

3.6 Summary

In this chapter, we have presented a centralized adaptive scheme for a MAS that aims to maintain its nominal collective behaviour subject to uncertain nonlinearities. The controller contains two main components that can be designed separately. The first is a control protocol designed for the MAS when free of uncertainties. The second is an adaptive controller added to the control structure when the nonlinear dynamics with unknown constant parameters exist. The proposed adaptive controller is incorporated by the certainty equivalence principle. The effectiveness of our approach is presented for the consensus problem for second-order MASs subject to uncertainties under a fixed undirected network. We present some simulations with various settings to illustrate the

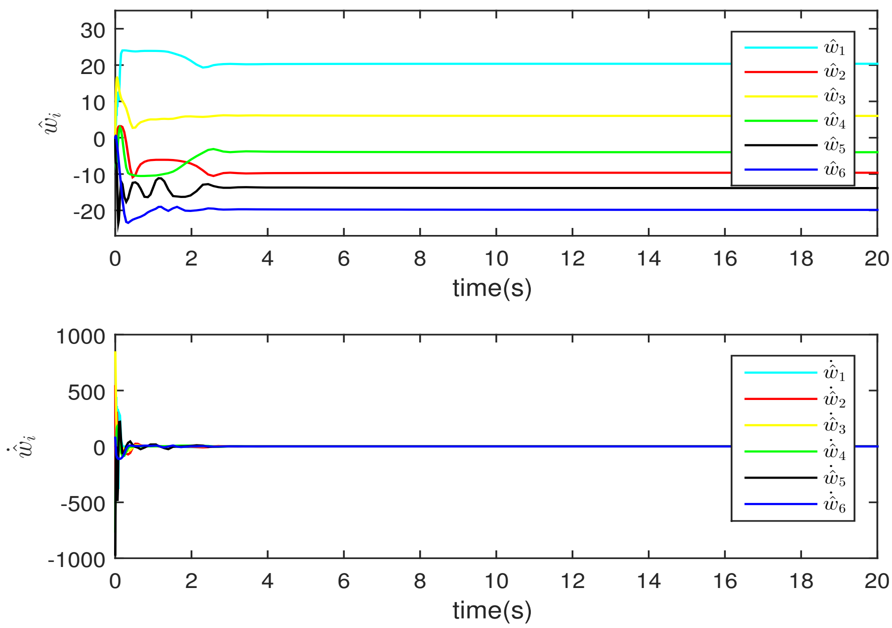

Figure 3.6: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ with adaptive controller

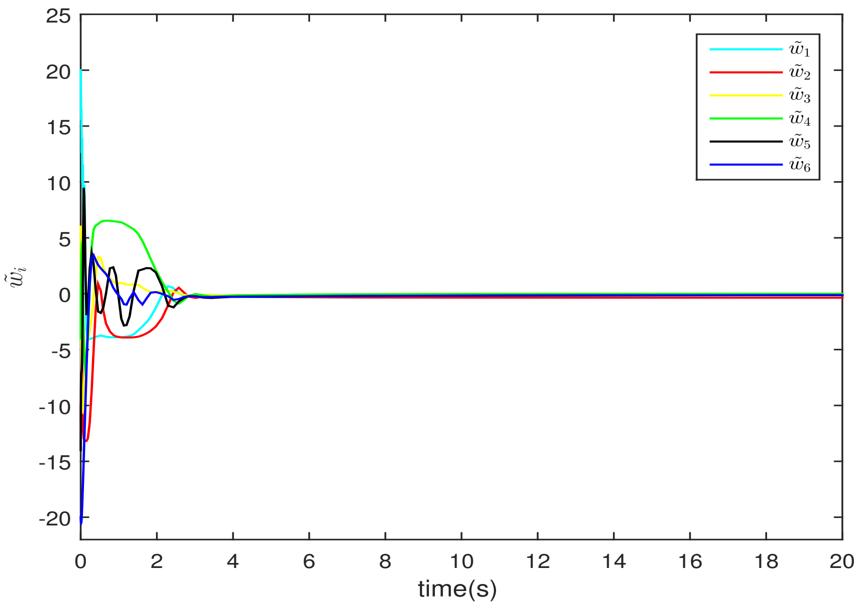

Figure 3.7: Profile of $\tilde{w}_i$ with adaptive controller

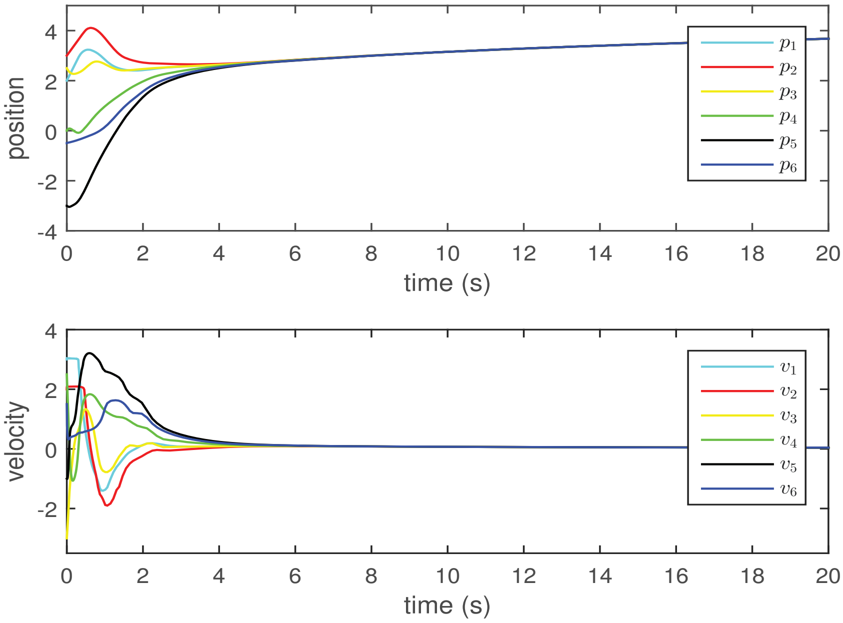

Figure 3.8: Profile of state trajectories of six agents with unknown constant parameters within interval $[-100, 100]$

performance of our control approach.

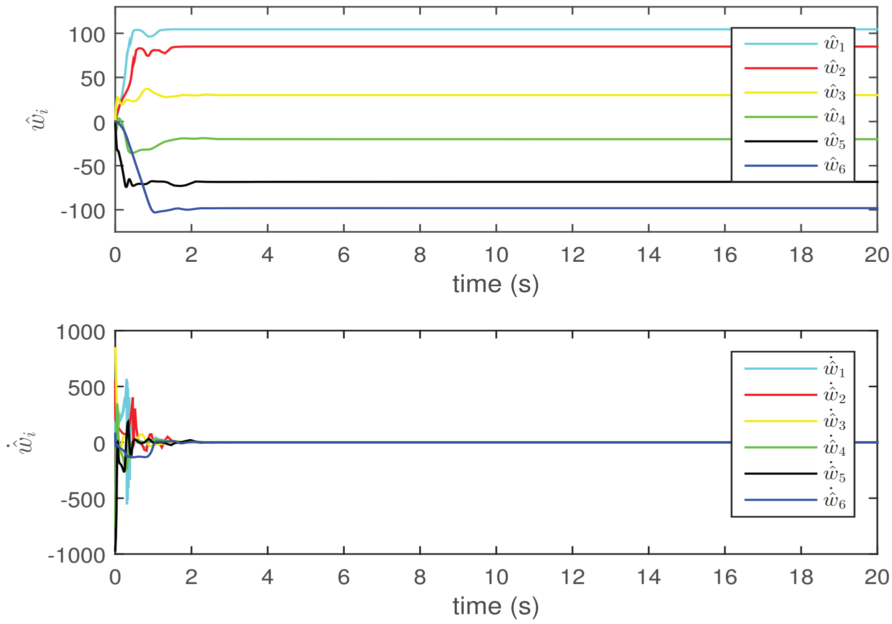

Figure 3.9: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ with adaptive controller and unknown constant parameters within interval $[-100, 100]$

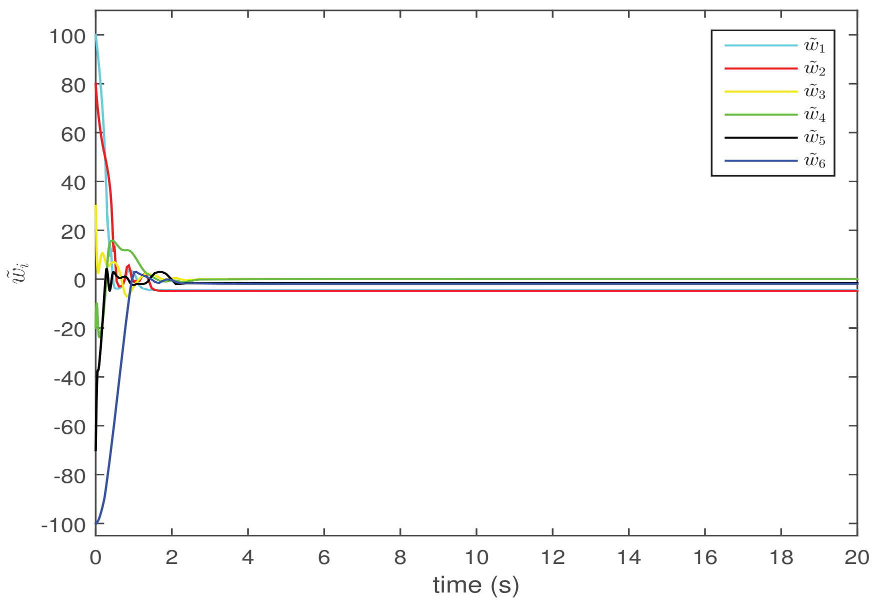

Figure 3.10: Profile of $\tilde{w}_i$ with adaptive controller and unknown constant parameters within interval $[-100, 100]$

4

Distributed Adaptive Consensus of Multi-Agent Systems with Linearly Parameterized Dynamics

4.1 Introduction

This chapter studies a distributed adaptive scheme for consensus of nonlinear MASs subject to uncertainties. As stated in Chapter 1, some results are available to deal with consensus of nonlinear MASs with uncertainties. However, it is still challenging to design distributed adaptive consensus control to achieve asymptotic consensus.

Recently, the adaptive control technique has been investigated to study consensus problems for MASs with uncertainties. For instance, a first-order MAS was first studied in [87] under an undirected communication graph, and a more general framework for a group of continuous-time systems was considered in [88]. A similar adaptive technique was used in [89] for both first and second-order MASs with a Nussbaum gain added to deal with unknown control direction. The undirected fixed topology has been extended to undirected jointly connected switching topology in [93] for leader-following case and in [94] for leaderless case.

In the above schemes, although the adaptive law along the gradient of a Lyapunov function is effective, it has the limitation that the technique cannot be generalized to handle second-order MASs with a directed graph. It essentially requires all the agent states to construct a global Lyapunov function. This intrinsic methodology limitation makes general distributed implementation difficult for tackling the consensus problem for complicated MASs.

For a directed graph, the associated Laplacian matrix is asymmetric, which significantly complicates the problem. Some relevant work for consensus of MASs with uncertainties can be found in [97], which gave a result for higher-order MASs, but for the leader-following case. Moreover, it is noted that consensus in [97] cannot be achieved asymptotically but with a residual error. The work in [97] also considers NN approximation for the unknown nonlinearities. The residual error is caused not only by NN approximation errors, but also by the distributed implementation of the adaptive law. In other words, residual consensus errors still exist even if the NN approximation error is zero. The work in [97] includes the early results in [95, 96] as special cases.

Even though an adaptive law along the gradient of Lyapunov function using the certainty equivalence principle has been proved to be successful in the aforementioned scenarios, it does not work for MASs in general, as a Lyapunov function is usually centrally constructed. In other words, distributed implementation of the gradient of Lyapunov function is usually impractical except for limited cases. For instance, it still remains open to design a distributed adaptive law to achieve asymptotic consensus for a second-order MAS in a directed network. As will be explained in detail in the next section, an adaptive law along the gradient of Lyapunov function has its inherent drawbacks for solving this open problem due to the lack of its distributed implementation.

In this chapter, we propose a novel distributed adaptive scheme, not relying on the gradient of Lyapunov function, for general nonlinear MASs with unknown constant parameters. In the gradient based scheme, the estimation error is expected to have a steady state zero. To drive the agent states together with the estimation error to their steady states, the adaptive law must follow the gradient of Lyapunov function. The novel idea is to introduce an input compensation such that the steady state of the estimation error is not zero but a manifold in the state space of agent states and estimated parameters. By proper selection of the manifold, it can be made attractive without relying on the centrally designed Lyapunov function. At the manifold, the agent states also approach their desired steady state. The idea in characterizing the steady-state manifold originates from the steady-state generator design in the output regulation theory for dealing with asymptotic disturbance rejection and reference tracking [124, 125] and immersion and invariance adaptive control of nonlinearly parameterized systems [115]. Within the novel distributed adaptive scheme, the aforementioned open problem on asymptotic consensus of a second-order nonlinear MAS in a directed network is solved.

4.2 Preliminaries and Motivating Examples

Recall a MAS with a properly designed controller (3.1), described by

$$\dot{x}_i = f_i(x),\ i = 1, \cdots, n \tag{4.1}$$

where $x_i \in \mathbb{R}^l$ is the state of the i-th agent and $x = [x_1^\top, x_2^\top, \cdots, x_n^\top]^\top$. Denote $f(x) = [f_i^\top(x_1), f_2^\top(x_2), \cdots, f_n^\top(x_n)]^\top$ and the network has the compact form $\dot{x} = f(x)$. This is the nominal closed-loop MAS free of uncertainties. Suppose the MAS has achieved a certain consensus behaviour, as described by a property in terms of a Lyapunov-like function.

Assumption 4.2.1 For systems 4.1. There exists a continuously differentiable function $V(x)$ satisfying

$$\underline{\alpha}(\|x\|_R) \leq V(x) \leq \bar{\alpha}(\|x\|_R)$$

for a matrix $R \in \mathbb{R}^{\bar{n}l \times nl}$ with $\bar{n} \leq n$ and class $\mathcal{K}_\infty$ functions $\underline{\alpha}$ and $\bar{\alpha}$, such that,

$$\frac{\partial V(x)}{\partial x} f(x) \leq -\alpha(\|x\|_R) \tag{4.2}$$

for a class $\mathcal{K}_\infty$ function α. Moreover,

$$\frac{\left\| \frac{\partial V(x)}{\partial x} \right\|^2}{\alpha(\|x\|_R)} \leq \sigma, \tag{4.3}$$

for some positive constant σ. ∎

Remark 4.2.1 Two typical scenarios of Assumption 4.2.1 are explained as follows.

(i) If $R \in \mathbb{R}^{nl \times nl}$, i.e., $\bar{n} = n$, is a nonsingular matrix, then $\|x\|_R = 0$ implies $\|x\| = 0$. In this scenario, the function $V(x)$ is a Lyapunov function for the x-system and Assumption 4.2.1 implies $\lim_{t\to\infty} \|x(t)\| = 0$, i.e., asymptotic stability about the equilibrium at the origin.

(ii) If $R \in \mathbb{R}^{(n-1)l \times nl}$, i.e., $\bar{n} = n - 1$, is a full row rank matrix and the rows are perpendicular to span$\{\mathbf{1} \otimes I_l\}$ where $I_l \in \mathbb{R}^{l\times l}$ is an identity matrix and $\mathbf{1} = \begin{bmatrix} 1 & \dots & 1 \end{bmatrix}^\intercal \in \mathbb{R}^n$, then $\|x\|_R = 0$ implies $x = \mathbf{1} \otimes x_o$ for some $x_o \in \mathbb{R}^l$. In this scenario, the function $V(x)$ is a Lyapunov function for the Rx-subsystem and Assumption 4.2.1 implies $\lim_{t\to\infty} \|x(t)\|_R = 0$, i.e., $\lim_{t\to\infty}[x(t) - \mathbf{1} \otimes x_o(t)] = 0$, which is a typical consensus phenomenon.

∎

Now, we consider the network subject to uncertainties. The objective is to design an adaptive scheme to deal with the uncertainties such that the behaviour of the nominal system is still maintained. The design of an adaptive law is expected to be separated from the consensus controller in the nominal system, which is not explicitly shown in the closed-loop structure (4.1).

Specifically, the network of MAS subject to uncertainties is represented by

$$\dot{x}_i = f_i(x) + g_i(x_i, w_i, \mu_i), \; i = 1, \cdots, n \tag{4.4}$$

where the nonlinear function $g_i(x_i, w_i, \mu_i) \in \mathbb{R}^l$ contains constant unknown parameters w_i and an additional adaptive control input μ_i to handle the uncertain nonlinear dynamics Suppose the uncertainties have the linearly parameterized structure, i.e.,

$$g_i(x_i, w_i, \mu_i) = h_i(x_i)(w_i - \mu_i). \tag{4.5}$$

We can rewrite the system in a compact form

$$\dot{x} = f(x) + H(x)(w - \mu), \tag{4.6}$$

where

$$\begin{aligned} w &= [w_1^\intercal, w_2^\intercal, \cdots, w_n^\intercal]^\intercal \\ \mu &= [\mu_1^\intercal, \mu_2^\intercal, \cdots, \mu_n^\intercal]^\intercal \\ H(x) &= \mathrm{diag}\begin{bmatrix} h_1(x_1) & h_2(x_2) & \cdots & h_n(x_n) \end{bmatrix}. \end{aligned}$$

If the parameter w_i were known, $\mu_i = w_i$ could trivially cancel the uncertainties $g_i(x_i, w_i, \mu_i)$. For the practical case with an unknown w_i, an adaptive law can be designed along the gradient of the Lyapunov function $V(x)$ as presented in Theorem 3.3.1. However, the adaptive law (3.10) is not always distributed, as $\partial V(x)/\partial x_i$ depends on not only the local state of agent i, but also the full network state x unless $V(x)$ can be properly designed to have a distributed $\partial V(x)/\partial x_i$ on a case by case basis. However, it can be true only for very limited cases because the function $V(x)$ for the nominal system is constructed in a centralized manner. Two motivating examples are given as follows.

Example 4.2.1 Consider a first-order integrator MAS in the network of an undirected graph associated with a symmetric Laplacian L. The nominal network dynamics are given as

$$\dot{x} = -Lx,$$

where $x \in \mathbb{R}^n$. The Lyapunov function of closed-loop system is chosen to be

$$V(x) = \frac{1}{2} x^\mathsf{T} L x,$$

where $L = R^\mathsf{T} R$ for a full row rank matrix $R \in \mathbb{R}^{(n-1)\times n}$ when the graph is connected.

The time-derivative of $V(x)$ along the trajectory of $\dot{x} = -Lx$ is

$$\begin{aligned} \dot{V}(x) &= -x^\mathsf{T} L L x \\ &= -(Rx)^\mathsf{T}(RR^\mathsf{T})(Rx) \\ &\leq -r_{\min}\|x\|_R^2, \end{aligned}$$

where $r_{\min} > 0$ is the minimal eigenvalue of RR^T. When the network is subject to uncertainties $H(x)w$, i.e., $\dot{x} = -Lx + H(x)(w-\mu)$, following Theorem 3.3.1, the additional adaptive controller μ in (3.7) has the specific form

$$\begin{aligned} \dot{\hat{w}}_i &= \lambda h_i^\mathsf{T}(x_i) L_i x \\ &= \lambda h_i^\mathsf{T}(x_i) \sum_{j\in\mathcal{N}_i\cup\{i\}} l_{ij} x_j, \ \lambda > 0, \end{aligned}$$

with L_i the i-th row of L and $\mathcal{N}_i$ the set of neighbours of i. In this scenario, the adaptive scheme is implemented in a distributed fashion. This development can be found in, e.g., [94]. ∎

The adaptive law (3.26) studied in Chapter 3 is another example showing that the adaptive law (3.10) can be implemented in a distributed fashion as a special case.

Example 4.2.2 Consider a first-order integrator MAS $\dot{x} = -Lx$ in the network of a directed graph associated with a Laplacian matrix of the special form

$$L = \begin{bmatrix} 0 & 0_{1\times(n-1)} \\ -b & L_o + B \end{bmatrix} \tag{4.7}$$

that represents a leader-following network with agent 1 as the leader. The matrix L_o is the Laplacian of the sub-network of followers and $B = \mathrm{diag}(b), b = [b_2, \cdots, b_n]^\intercal$ with $b_i \geq 0$ the weight from the leader to agent i. Denote $L_o = D - E$ where $D = \mathrm{diag}(d_2, \cdots, d_n)$ a diagonal matrix and E an off-diagonal one. Assume the network has a spanning tree with the root node being the leader node 1. Then, there exists a diagonal matrix $P = \mathrm{diag}(p_2, \cdots, p_n) > 0$ such that

$$2Q = P(L_o + B) + (L_o + B)^\intercal P > 0.$$

Let

$$R = \begin{bmatrix} -b & L_o + B \end{bmatrix},$$

one has

$$RL = (L_o + B)R.$$

The time-derivative of Lyapunov function

$$V(x) = \frac{1}{2} x^\intercal R^\intercal P R x$$

along the trajectory of $\dot{x} = -Lx$ is

$$\begin{aligned} \dot{V}(x) &= -\frac{1}{2} x^\intercal R^\intercal [P(L_o + B) + (L_o + B)^\intercal P] R x \\ &\leq -x^\intercal R^\intercal Q R x. \end{aligned}$$

When the network is subject to uncertainties $H(x)w$, i.e., $\dot{x} = -Lx + H(x)(w - \mu)$, along which the time-derivative of

$$U(x, \tilde{w}) = V(x) + \frac{1}{2\lambda} \tilde{w}^\intercal \tilde{w}$$

is

$$\dot{U}(x) \leq -x^\intercal R^\intercal Q R x + [\frac{1}{\lambda} \dot{\hat{w}}^\intercal - x^\intercal R^\intercal P R H(x)] \tilde{w}.$$

Following Theorem 3.3.1, the update law in (3.7) has the specific form

$$\dot{\hat{w}} = \lambda H^{\intercal}(x) R^{\intercal} P R x. \tag{4.8}$$

The update law (4.8) shows that not only is the information about neighbours required to generate the update law, but also the information about the neighbour's neighbours. It means that the update law (4.8) cannot be implemented in distributed fashion. In fact, a distributed adaptive law for this scenario still remains open.

For the scenario studied in [97], the leader is free of uncertainty, i.e., $h_1(x) = 0$ and $w \in \mathbb{R}^0$ trivially. Then, one has

$$\begin{aligned} RH(x) &= (L_o + B)\mathrm{diag}\begin{bmatrix} 0 & h_2(x_2) & \cdots & h_n(x_n) \end{bmatrix} \\ &= (L_o + B)\bar{H}(x) \\ &= (D + B)\bar{H}(x) - E\bar{H}(x) \end{aligned}$$

for $\bar{H}(x) = \Big[0, \mathrm{diag}\begin{bmatrix} h_2(x_2) & \cdots & h_n(x_n) \end{bmatrix}\Big]$. The following update law was applied

$$\dot{\hat{w}} = \lambda \bar{H}^{\intercal}(x)(D + B) P R x - \lambda\kappa\hat{w},$$

that gives

$$\begin{aligned} \dot{U}(x) &\leq -x^{\intercal} R^{\intercal} Q R x + [-\kappa\hat{w}^{\intercal} + x^{\intercal} R^{\intercal} P E \bar{H}(x)]\tilde{w} \\ &\leq \left[-x^{\intercal} R^{\intercal} Q R x + x^{\intercal} R^{\intercal} P E \bar{H}(x)\tilde{w} - \kappa\|\tilde{w}\|^2\right] + \kappa\|w\|\|\tilde{w}\|. \end{aligned}$$

The update law is implemented in a distributed fashion by noting that the matrices P, D and B are diagonal, that is,

$$\dot{\hat{w}}_i = \lambda(d_i + b_i) p_i \bar{h}_i^{\intercal}(x) L_i x - \lambda\kappa\hat{w}_i, \ , i = 2, \cdots, n,$$

with L_i the i-th row of L. In the expression of $\dot{U}(x)$, the terms in the square brackets can be made negative with a sufficiently large κ but the positive term $\kappa\|w\|\|\tilde{w}\|$ causes a residual consensus error. In other words, no asymptotic consensus can be achieved using the approach developed in [97]. ∎

4.3 A Distributed Adaptive Scheme

The main contribution of our approach in this chapter is to bring a novel adaptive scheme that can be implemented in a distributed fashion. For this purpose, let us have a close inspection of the approach in Theorem 3.3.1. For the system (4.4) with linearly parameterized uncertainties, we introduce a virtual exosystem

$$\begin{aligned}\dot{\tau}_i &= f_i(\tau) \\ \dot{w}_i &= 0,\ i = 1, \cdots, n.\end{aligned} \tag{4.9}$$

The agent state x_i and input μ_i are expected to have the steady states $x_{i,ss} = \tau_i$ and $\mu_{i,ss} = w_i$, respectively. In this sense, we call

$$\begin{aligned}\dot{w}_i &= 0, \\ \mu_{i,ss} &= w_i,\ i = 1, \cdots, n\end{aligned}$$

the steady-state generator for the input μ_i, which motivates the update law

$$\begin{aligned}\dot{\hat{w}}_i &= 0 + \nabla, \\ \mu_i &= \hat{w}_i,\ i = 1, \cdots, n,\end{aligned}$$

where ∇ is designed along the gradient of Lyapunov function such that the manifold $\{(x, \mu, \tau, w) \mid x_i = \tau_i,\ \mu_i = w_i,\ i = 1, \cdots, n\}$ is attractive.

The novel idea is to introduce a function $\beta_i(x_i)$ to the input, i.e., $\mu_i = -\beta_i(x_i) + \hat{\mu}_i$. Along the virtual exosystem (4.9), the agent state x_i and input $\hat{\mu}_i$ are expected to have the steady states $x_{i,ss} = \tau_i$ and $\hat{\mu}_{i,ss} = \theta_i(\tau_i, w_i) = \beta_i(\tau_i) + w_i$, respectively. As a result, we have a steady-state generator for the input $\hat{\mu}_i$

$$\begin{aligned}\dot{\theta}_i(\tau_i, w_i) &= \frac{\partial \beta_i(\tau_i)}{\partial \tau_i} f_i(\tau) \\ \hat{\mu}_{i,ss} &= \theta_i(\tau_i, w_i),\ i = 1, \cdots, n,\end{aligned}$$

that motivates the update law

$$\begin{aligned}\dot{\hat{w}}_i &= \frac{\partial \beta_i(x_i)}{\partial x_i} f_i(x) \\ \hat{\mu}_i &= \hat{w}_i,\ \ i = 1, \cdots, n.\end{aligned}$$

In this design, β_i can be properly selected such that the manifold $\{(x, \hat{\mu}, \tau, w) \mid x_i = \tau_i,\ \hat{\mu}_i = \theta_i(\tau_i, w_i),\ i = 1, \cdots, n\}$ is attractive. The introduction of β_i avoids the implementation of ∇ that relies on a centrally designed Lyapunov function.

In this new development, if we treat $\hat{w}_i$ as the estimated value of w_i, the steady state of the estimation error $\hat{w}_i - w_i$ is not zero but $\theta_i(\tau_i, w_i) - w_i = \beta_i(\tau_i)$ where τ_i is the steady state of x_i. Therefore, we aim to drive $\hat{w}_i - w_i$ to the manifold $\{(x_i, \hat{w}_i) \mid \hat{w}_i - w_i = \beta_i(x_i),\ i = 1, \cdots, n\}$ in the space of agent states and estimated parameters. By proper selection of the manifold, it can be made attractive and the agent state x_i can approach its desired steady state τ_i on the manifold. The rigorous formulation of the approach is given in the following theorem.

Theorem 4.3.1 *Consider the system (4.4) with (4.5) under Assumption 4.2.1. Let the distributed controller be*

$$\begin{aligned} \mu_i &= \hat{w}_i - \beta_i(x_i) \\ \dot{\hat{w}}_i &= -\lambda_i h_i^{\mathsf{T}}(x_i) f_i(x) \end{aligned} \tag{4.10}$$

where $\beta_i(x_i)$ is any continuously differentiable function satisfying

$$\frac{\partial \beta_i(x_i)}{\partial x_i} = -\lambda_i h_i^{\mathsf{T}}(x_i), \tag{4.11}$$

for some $\lambda_i > 0$. Then the time-derivative of

$$U(x, z) = V(x) + \frac{\sigma}{4(1-k)} \sum_{i=1}^{n} \frac{1}{2\lambda_i} z_i^{\mathsf{T}} z_i, \tag{4.12}$$

with

$$z_i = \beta_i(x_i) - \tilde{w}_i,\ \tilde{w}_i = \hat{w}_i - w_i, \tag{4.13}$$

satisfies

$$\dot{U}(x, z) \le -k\alpha(\|x\|_R), \tag{4.14}$$

for any $0 < k < 1$, along the trajectory of the closed-loop system (4.4)+(4.5)+(4.10). ∎

Proof: The system composed of (4.4)+(4.5)+(4.10) can be rewritten as

$$\begin{aligned} \dot{x}_i &= f_i(x) + h_i(x_i, w_i) - h_i(x_i, \hat{w}_i - \beta_i(x_i)) \\ &= f_i(x) + h_i(x_i, w_i) - h_i(x_i, w_i - z_i). \end{aligned}$$

Direct calculation shows

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V(x)}{\partial x} f(x) + \sum_{i=1}^{n} \frac{\partial V(x)}{\partial x_i} h_i(x_i) z_i \\ &\le -\alpha(\|x\|_R) + \sum_{i=1}^{n} \frac{\partial V(x)}{\partial x_i} h_i(x_i) z_i. \end{aligned}$$

For any $0 < k < 1$, pick $a = (1-k)/\sigma$. One has

$$a \left\| \frac{\partial V(x)}{\partial x} \right\|^2 \leq (1-k)\alpha(\|x\|_R).$$

Moreover

$$\begin{aligned}
\dot{V}(x) &\leq -\alpha(\|x\|_R) + \sum_{i=1}^{n} \left\{ a \left\| \frac{\partial V(x)}{\partial x_i} \right\|^2 + \frac{1}{4a}\|h_i(x_i)z_i\|^2 \right\} \\
&\leq -\alpha(\|x\|_R) + a \left\| \frac{\partial V(x)}{\partial x} \right\|^2 + \sum_{i=1}^{n} \frac{1}{4a}\|h_i(x_i)z_i\|^2 \\
&\leq -k\alpha(\|x\|_R) + \sum_{i=1}^{n} \frac{1}{4a}\|h_i(x_i)z_i\|^2.
\end{aligned}$$

Next, one has

$$\begin{aligned}
\dot{z}_i &= \frac{\partial \beta_i(x_i)}{\partial x_i}\dot{x}_i - \dot{\hat{w}}_i \\
&= \frac{\partial \beta_i(x_i)}{\partial x_i} f_i(x) + \frac{\partial \beta_i(x_i)}{\partial x_i} h_i(x_i) z_i - \frac{\partial \beta_i(x_i)}{\partial x_i} f_i(x) \\
&= \frac{\partial \beta_i(x_i)}{\partial x_i} h_i(x_i) z_i \\
&= -\lambda_i h_i^{\mathsf{T}}(x_i) h_i(x_i) z_i.
\end{aligned}$$

Then, the derivative of

$$\frac{1}{2\lambda_i} z_i^{\mathsf{T}} z_i$$

along the above trajectory is

$$\begin{aligned}
\frac{d(\frac{1}{2\lambda_i} z_i^{\mathsf{T}} z_i)}{dt} &= -z_i^{\mathsf{T}} h_i^{\mathsf{T}}(x_i) h_i(x_i) z_i \\
&= -\|h_i(x_i)z_i\|^2.
\end{aligned}$$

As a result, the derivative of

$$U(x,z) = V(x) + \frac{1}{4a}\sum_{i=1}^{n} \frac{1}{2\lambda_i} z_i^{\mathsf{T}} z_i,$$

along the trajectory of the closed-loop system is

$$\begin{aligned}
\dot{U}(x,z) &\leq -k\alpha(\|x\|_R) + \sum_{i=1}^{n} \frac{1}{4a}\|h_i(x_i)z_i\|^2 - \sum_{i=1}^{n} \frac{1}{4a}\|h_i(x_i)z_i\|^2 \\
&\leq -k\alpha(\|x\|_R).
\end{aligned}$$

The proof is thus completed. ∎

Remark 4.3.1 In Theorem 4.3.1 the adaptive controller (4.10) is implemented at each agent i. This scheme is distributed as it only relies on the agent state x_i and its nominal dynamics $f_i(x)$. The nominal dynamics $f_i(x)$ is implemented beforehand for the ideal situation free of uncertainties, typically in distributed fashion. The effectiveness of Theorem 4.3.1 will be demonstrated by a network of second-order uncertain dynamics in the next section. ∎

4.4 Application to A Network of Second-Order Uncertain Dynamics

We consider a group of $n \geq 2$ agents governed by a set of second-order nonlinear differential equations

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \alpha_1 p_i + \alpha_2 v_i + \xi_i\left(w_i, v_i\right) + u_i, \ i = 1, \ldots, n, \end{aligned} \tag{4.15}$$

where $p_i, v_i \in \mathbb{R}$ are the states and $u_i \in \mathbb{R}$ is the input of the agent i. The function $\xi_i(w_i, v_i) = \zeta_i(v_i) w_i$ for a bounded function $\zeta_i(v_i)$ represents heterogeneous nonlinearities with w_i an unknown constant parameter. The two parameters α_1 and α_2 are known. For convenience of presentation, we denote

$$A = \begin{bmatrix} 0 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix}, \quad x_i = \begin{bmatrix} p_i \\ v_i \end{bmatrix}$$

and

$$p = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}.$$

In this section, the network of MASs is given by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, \cdots, n\}$ is a finite non-empty set of nodes (i.e., agents) and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of edges (i.e., communication links). The adjacency matrix $\mathcal{A} = [a_{ij}]$ of a weighted directed graph is defined as $a_{ii} = 0$ (no self-loop) and $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ where $i \neq j$. The Laplacian L has elements of $L_{ii} = \sum_{j \neq i} a_{ij}$ and $L_{ij} = -a_{ij}$, where $i \neq j$. For a distributed algorithm, each agent i can achieve the information from the network as follows, with L_i denoting the i-th row of L,

$$\begin{aligned} L_i p &= -\sum_{j=1}^{n} a_{ij}(p_j - p_i) \\ L_i v &= -\sum_{j=1}^{n} a_{ij}(v_j - v_i). \end{aligned}$$

In this section, we study a general directed leaderless setting that includes the leader-following case (with the Laplacian of the special form (4.7)) as a special case. Throughout the section, we have the following assumption.

Assumption 4.4.1 The network has a directed spanning tree. ∎

The objective is to design a distributed adaptive consensus protocol (i.e., only p_i, v_i, $L_i p$ and $L_i v$ are available measurements for agent i) under which the MAS under Assumption 4.4.1 has the following asymptotic property

$$\begin{aligned} \lim_{t\to\infty} p(t) - p_o(t)\mathbf{1} &= 0 \\ \lim_{t\to\infty} v(t) - v_o(t)\mathbf{1} &= 0 \end{aligned} \tag{4.16}$$

for some time functions $p_o(t), v_o(t) : [0, \infty) \mapsto \mathbb{R}$.

Under Assumption 4.4.1, the Laplacian L has one zero eigenvalue and the remaining eigenvalues contain positive real parts. Let the vectors $r \in \mathbb{R}^n$ and $\mathbf{1}$ be the left and right eigenvectors corresponding to the eigenvalue zero of L, in particular, $r^\intercal L = 0$, $L\mathbf{1} = 0$, and $r^\intercal \mathbf{1} = 1$.

There exist matrices $W \in \mathbb{R}^{(n-1)\times n}$, $U \in \mathbb{R}^{n\times(n-1)}$ such that

$$T = \begin{bmatrix} r^\intercal \\ W \end{bmatrix}, \ T^{-1} = \begin{bmatrix} \mathbf{1} & U \end{bmatrix}.$$

One has the following similarity transformation

$$TLT^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}$$

where $J = WLU \in \mathbb{R}^{(n-1)\times(n-1)}$ is a matrix with all eigenvalues having positive real parts. Define the matrix R as follows

$$\begin{bmatrix} Wp \\ Wv \end{bmatrix} = Rx.$$

It is easy to check that R has a full row rank and the rows of R are perpendicular to $\mathrm{span}\{\mathbf{1} \otimes I_2\}$.

We first have the following technical lemma.

Lemma 4.4.1 *Under Assumption 4.4.1, there exist $\gamma_1, \gamma_2 > 0$ such that the matrix*

$$\bar{A} = \begin{bmatrix} 0 & I \\ \alpha_1 I - \gamma_1 J & \alpha_2 I - \gamma_2 J \end{bmatrix}$$

is Hurwitz. ▮

Proof: Under Assumption 4.4.1, all eigenvalues of J have positive real parts. Let $P_J \in \mathbb{R}^{(n-1)\times(n-1)}$ be the positive definite matrix such that

$$P_J J + J^\intercal P_J = I.$$

Let c be a positive constant such that

$$P_J < 2cI,$$

which, by Schur complement, implies

$$Q = \begin{bmatrix} -I & P_J - cI \\ P_J - cI & -c^2 I \end{bmatrix} < 0. \tag{4.17}$$

By choosing $\gamma_2 = c\gamma_1$ and a sufficiently large $\gamma_1 > 0$, we will show $\bar{A}$ is Hurwitz. Denote

$$P = \begin{bmatrix} \gamma_1 P_J & P_J \\ P_J & cP_J \end{bmatrix},$$

which is positive definite if $c\gamma_1 > 1$. Note that

$$P\bar{A} + \bar{A}^\intercal P = \gamma_1 Q + Q_c,$$

where

$$Q_c = \begin{bmatrix} 2\alpha_1 P_J & (\alpha_2 + \alpha_1 c)P_J \\ (\alpha_2 + \alpha_1 c)P_J & 2(1 + \alpha_2 c)P_J \end{bmatrix}$$

is a constant matrix. For a sufficiently large γ_1, $P\bar{A} + \bar{A}^\intercal P = \gamma_1 Q + Q_c < 0$ due to (4.17). Therefore, $\bar{A}$ is Hurwitz. ■

The next lemma shows the consensus result for the ideal situation.

Lemma 4.4.2 *Under Assumption 4.4.1, consider the system (4.15) with* $\xi_i(w_i, v_i) = 0$ *and*

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v, \tag{4.18}$$

where γ_1 *and* γ_2 *are such that the matrix* $\bar{A} = \begin{bmatrix} 0 & I \\ \alpha_1 I - \gamma_1 J & \alpha_2 I - \gamma_2 J \end{bmatrix}$ *is Hurwitz. Let* $P = P^\mathsf{T} > 0$ *be the solution to the Lyapunov equation*

$$P\bar{A} + \bar{A}^\mathsf{T} P = -I.$$

The function

$$V(x) = x^\mathsf{T} R^\mathsf{T} P R x \tag{4.19}$$

satisfies $P_{\min}\|x\|_R^2 \le V(x) \le P_{\max}\|x\|_R^2$ *(*$P_{\min}$ *and* $P_{\max}$ *are the minimum and maximum eigenvalues of* P*) and its derivative along the closed-loop system is*

$$\dot{V}(x) = -\|x\|_R^2.$$

∎

Proof: The closed-loop system composed of (4.15) and (4.18) is

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v, \ i = 1, \dots, n, \end{aligned} \tag{4.20}$$

denoted as $\dot{x}_i = f_i(x)$. It can also be put in a compact form

$$\begin{aligned} \dot{p} &= v \\ \dot{v} &= \alpha_1 p + \alpha_2 v - \gamma_1 L p - \gamma_2 L v. \end{aligned} \tag{4.21}$$

From the definition of T and T^{-1}, one has

$$\mathbf{1} r^\mathsf{T} + UW = I.$$

and

$$\begin{aligned} JW &= WLUW \\ &= WL(I - \mathbf{1} r^\mathsf{T}) \\ &= WL. \end{aligned}$$

Using this fact, we have the following calculation

$$\begin{aligned}
R\dot{x} &= \begin{bmatrix} W\dot{p} \\ W\dot{v} \end{bmatrix} \\
&= \begin{bmatrix} Wv \\ \alpha_1 Wp + \alpha_2 Wv - \gamma_1 WLp - \gamma_2 WLv \end{bmatrix} \\
&= \begin{bmatrix} Wv \\ \alpha_1 Wp + \alpha_2 Wv - \gamma_1 JWp - \gamma_2 JWv \end{bmatrix} \\
&= \begin{bmatrix} 0 & I \\ \alpha_1 I - \gamma_1 J & \alpha_2 I - \gamma_2 J \end{bmatrix} \begin{bmatrix} Wp \\ Wv \end{bmatrix} \\
&= \bar{A} R x.
\end{aligned}$$

As a result,

$$\begin{aligned}
\dot{V}(x) &= x^{\mathsf{T}} R^{\mathsf{T}} P R \dot{x} + \dot{x}^{\mathsf{T}} R^{\mathsf{T}} P R x \\
&= x^{\mathsf{T}} R^{\mathsf{T}} (P\bar{A} + \bar{A}^{\mathsf{T}} P) R x \\
&= -\|x\|_R^2.
\end{aligned}$$

The proof is completed. ∎

The main result on a distributed adaptive controller is stated in the following theorem that is proved by applying Theorem 4.3.1.

Theorem 4.4.1 *Under Assumption 4.4.1, consider the system (4.15) with the controller*

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v - \zeta_i(v_i)\mu_i, \tag{4.22}$$

where γ_1 and γ_2 are given in Lemma 4.4.2,

$$\begin{aligned}
\mu_i &= \hat{w}_i - \rho_i(v_i) \\
\dot{\hat{w}}_i &= -\lambda_i \zeta_i^{\mathsf{T}}(v_i)[\alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v],
\end{aligned} \tag{4.23}$$

and $\rho_i(v_i)$ is any continuously differentiable function satisfying

$$\frac{\partial \rho_i(v_i)}{\partial v_i} = -\lambda_i \zeta_i^{\mathsf{T}}(v_i), \ \lambda_i > 0. \tag{4.24}$$

Then, consensus is achieved in the sense of (4.16) for some time functions $p_o(t), v_o(t) : [0,\infty) \mapsto \mathbb{R}$. ∎

Proof: The closed-loop system composed of (4.15) and (4.22) is, for $i = 1, \dots, n$,

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v + \zeta_i(v_i)(w_i - \mu_i), \end{aligned} \tag{4.25}$$

or in a compact form (4.4), i.e.,

$$\dot{x}_i = f_i(x) + h_i(x_i)(w_i - \mu_i),$$

where $\dot{x}_i = f_i(x)$ is given in (4.20) and

$$h_i(x_i) = \left[\begin{array}{c} 0 \\ \zeta_i(v_i) \end{array} \right].$$

In Lemma 4.4.2, it has been proved that Assumption 4.2.1 is satisfied for $\dot{x}_i = f_i(x)$. It is noted that

$$\frac{\left\| \frac{\partial V(x)}{\partial x} \right\|^2}{\|x\|_R^2} = \frac{\|2x^\intercal R^\intercal P R\|^2}{\|x\|_R^2} \leq 4\|PR\|^2 < \infty. \tag{4.26}$$

For (4.24) and $\beta_i(x_i) = \rho_i(v_i)$, one has (4.11). Also, (4.10) takes the special form (4.23). By Theorem 4.3.1, one has

$$\dot{U}(x, z) \leq -k\|x\|_R^2 \tag{4.27}$$

for

$$U(x, z) = V(x) + \frac{\sigma}{4(1-k)} \sum_{i=1}^{n} \frac{1}{2\lambda_i} z_i^\intercal z_i,$$

and $z_i = \rho_i(v_i) - \tilde{w}_i$, $\tilde{w} = \hat{w} - w$.

It is obvious to see that both $\|x(t)\|_R$ and $z(t)$ are bounded. Because of

$$R\dot{x} = \bar{A}Rx + RH(x)z,$$

$\|\dot{x}(t)\|_R$ is bounded and hence $-k\|x(t)\|_R^2$ is uniformly continuous in t. By Barbalat's Lemma, one has $\lim_{t\to\infty} \|x(t)\|_R = 0$, that is,

$$\lim_{t\to\infty} \left[\begin{array}{c} Wp(t) \\ Wv(t) \end{array} \right] = 0.$$

Let $p_o(t) = r^\intercal p(t)$ and $v_o(t) = r^\intercal v(t)$. From the following relationship

$$\begin{aligned} p &= \left[\begin{array}{cc} \mathbf{1} & U \end{array} \right] \left[\begin{array}{c} r^\intercal p \\ Wp \end{array} \right] \\ &= \mathbf{1}(r^\intercal p) + U(Wp), \end{aligned}$$

and

$$\begin{aligned} v &= \left[\begin{array}{cc} \mathbf{1} & U \end{array}\right] \left[\begin{array}{c} r^{\mathsf{T}} v \\ W v \end{array}\right] \\ &= \mathbf{1}(r^{\mathsf{T}} v) + U(W v), \end{aligned}$$

one has

$$\begin{aligned} \lim_{t\to\infty} p(t) - p_o(t)\mathbf{1} &= U \lim_{t\to\infty} W p(t) = 0 \\ \lim_{t\to\infty} v(t) - v_o(t)\mathbf{1} &= U \lim_{t\to\infty} W v(t) = 0. \end{aligned}$$

The proof is thus completed. ■

Remark 4.4.1 The controller (4.22) consists of two components. The first component is designed as (4.18) for the ideal case with $\xi_i(w_i, v_i) = 0$ to achieve consensus. When the uncertainty $\xi_i(w_i, v_i)$ is taken into account, an additional adaptive compensator $-\zeta_i(v_i)\mu_i$ with the update law (4.23) is added to the controller. The critical advantage of the approach based on Theorem 4.4.1 is that the aforementioned two components can be designed separately. ▮

4.5 Numerical Simulation

We consider a network of $n = 6$ agents described by

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= -p_i + \xi_i(w_i, v_i) + u_i,\ i = 1, \ldots, n. \end{aligned} \tag{4.28}$$

The nonlinear uncertain terms $\xi_i\left(w_i, v_i\right)$'s are given as follows

$$\xi_i(w_i, v_i) = \begin{cases} w_i v_i^3, & i = 1, 2, 3 \\ w_i, & i = 4, 5 \\ w_i v_i, & i = 6 \end{cases}$$

Assume all the unknown parameters are selected within the interval $[-5, 5]$. The network topology is given in Fig. 4.1 with communication weights marked associated with the edges.

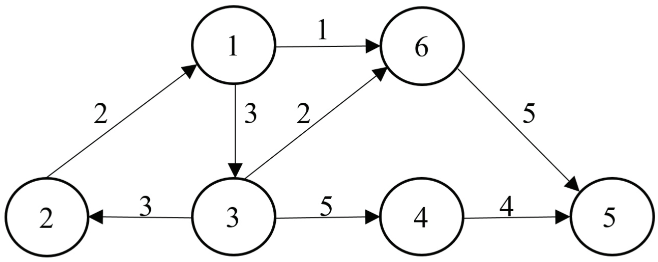

Figure 4.1: The network topology of six agents

From Fig. 4.1, we can generate the Laplacian matrix

$$L = \begin{bmatrix} 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 3 & -3 & 0 & 0 & 0 \\ -3 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & -5 & 5 & 0 & 0 \\ 0 & 0 & 0 & -4 & 9 & -5 \\ -1 & 0 & -2 & 0 & 0 & 3 \end{bmatrix}. \tag{4.29}$$

In this section, we illustrate the performance of theoretical results developed with several scenarios. The initial conditions are selected as follows

$$\begin{aligned} p(0) &= \begin{bmatrix} 3 & 5 & 2 & 0 & -5 & -3 \end{bmatrix}^{\mathsf{T}} \\ v(0) &= \begin{bmatrix} 5 & 1 & -5 & 3 & -2 & -1 \end{bmatrix}^{\mathsf{T}}, \end{aligned}$$

and the unknown constant parameters are

$$w = \begin{bmatrix} 5 & 4 & 0.5 & -0.5 & -1 & -5 \end{bmatrix}^{\mathsf{T}}.$$

The states of MAS (4.28) are expected to achieve consensus on a sinusoidal trajectory determined by the following nominal dynamics

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= -p_i. \end{aligned}$$

In the first scenario, we demonstrate the simulation of MAS (4.28) with $u_i = 0$. The information from the network and agent states is unavailable for feedback control for any agent. Consensus is not achieved under this situation, as illustrated in Fig. 4.2. We can see that agent i moves from its initial position and velocity with unstable velocity.

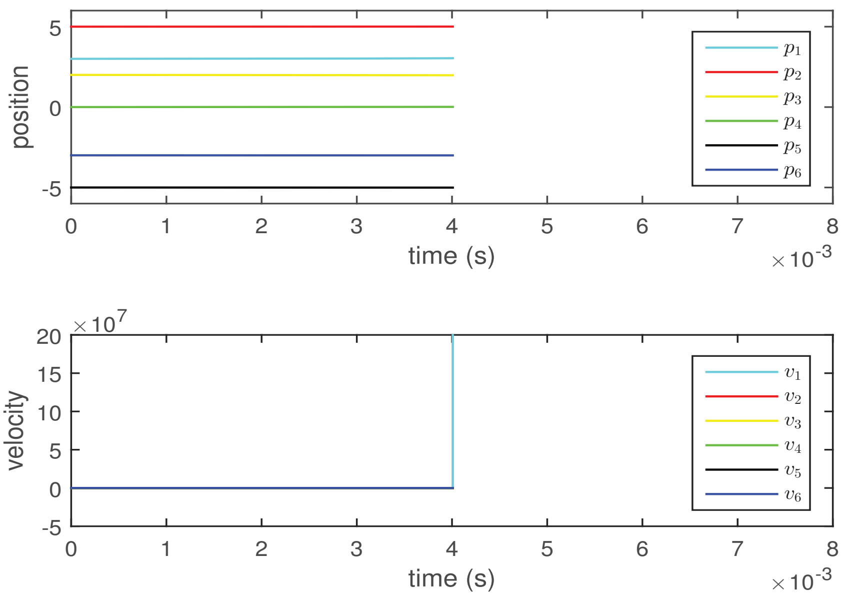

Figure 4.2: Profile of state trajectories of six agents without controller

The nominal behaviour of the agent i cannot be seen in this simulation, due to the presence of nonlinear dynamics.

Now, MAS (4.15) is considered to be free of uncertainties i.e. $\xi_i(w_i, v_i) = 0$. By Lemma 4.4.1, we can choose $\gamma_1 = 10$ and $\gamma_2 = 5$ such that $\bar{A}$ is Hurwitz. According to Lemma 4.4.2, consensus with a collective nominal dynamics (4.30) is guaranteed to be achieved under controller (4.18). Or, in other words, this situation represents MAS under the ideal condition $\dot{x} = f(x)$. The simulation results for this setting is plotted in Fig. 4.3. We can see that both position and velocity of every agent converge to a collective nominal behaviour i.e. sinusoidal trajectory.

When the uncertain nonlinearities $\xi_i(w_i, v_i)$ exist in the closed-loop systems, then an additional controller is required to be added in the control structure. The consensus controller developed in Theorem 4.4.1 is proposed in this scenario. The design of the controller (4.22) and (4.23) with $\rho_i(v_i)$ are specified as follows

$$\rho_i(v_i) = \begin{cases} -\lambda_i v_i^4/4, & i = 1,2,3 \\ -\lambda_i v_i, & i = 4,5 \\ -\lambda_i v_i^2/2, & i = 6 \end{cases}, \tag{4.30}$$

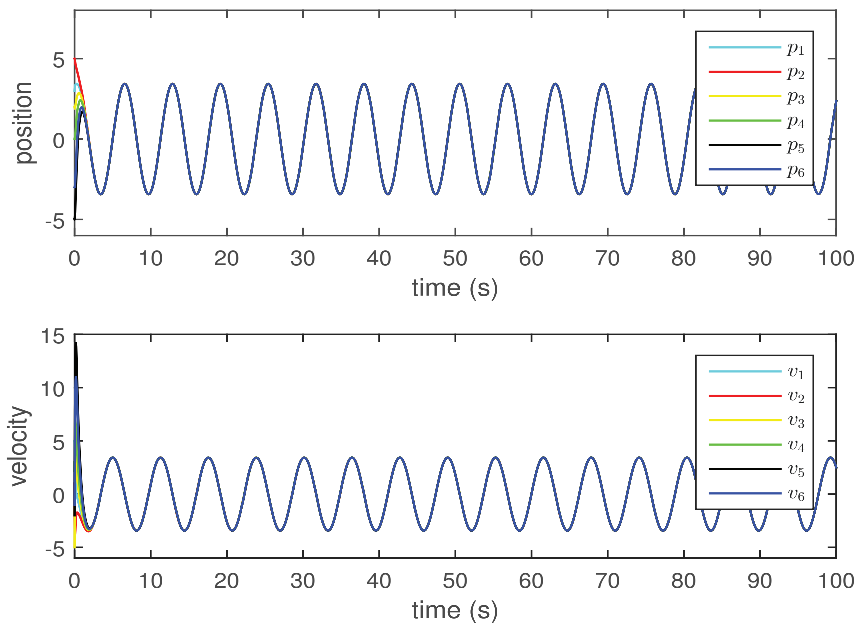

Figure 4.3: Profile of state trajectories of six agents under ideal situation

where $\lambda_i = 0.01$.

To illustrate the performance and the effectiveness of consensus control, we compare the response of the closed-loop system with and without adaptive controller. Fig. 4.4 and 4.5 illustrate the profile of state trajectories of six agents to achieve consensus without and with adaptive controller respectively. From the simulation results, we can see that consensus cannot be achieved for MAS without adaptive controller. The linear consensus protocol (4.18) is not enough to handle nonlinear dynamics. By adding adaptive controller, the uncertain nonlinearities can be handled. The profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ are illustrated in Fig. 4.6. Different to traditional adaptive controller, w_i doesn't converge to the actual value of w_i, but to $w_i + \rho_i(v_i)$ for deliberately designed $\rho_i(v_i)$. The state of z_i can be seen in Fig. 4.7. Asymptotic consensus is achieved as concluded in Theorem 4.4.1.

In the last scenario, the range of unknown constant parameters is increased to be within the interval $[-50, 50]$. In this simulation, w_i is selected as follows

$$w = \begin{bmatrix} 50 & 40 & 5 & -5 & -10 & -50 \end{bmatrix}^\top.$$

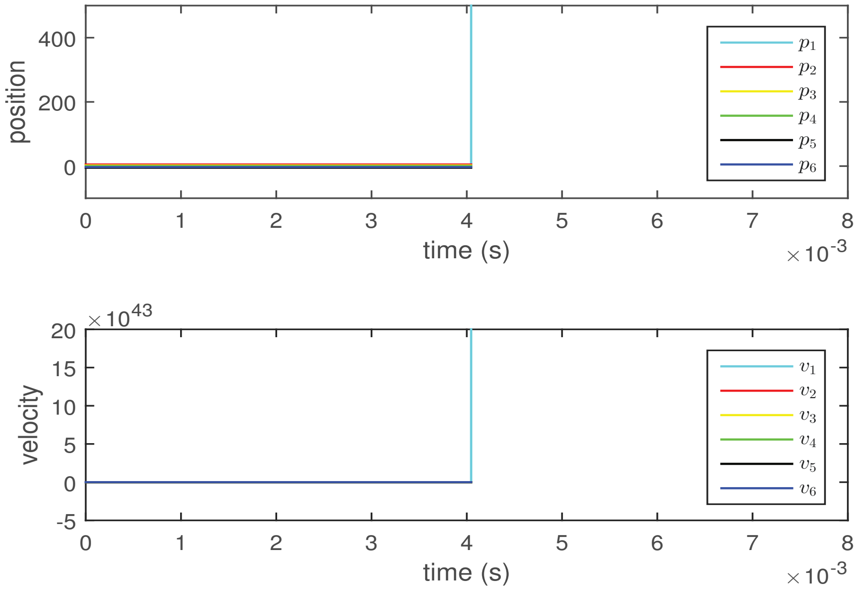

Figure 4.4: Profile of state trajectories of six agents without adaptive controller

Under the similar controller with the previous scenario, consensus still can be achieved by maintaining a collective nominal behaviour of the agents. Fig. 4.8 shows the profile of position and velocity consensus of six agents. Compared with the case where there were unknown constant parameters within interval $[-5, 5]$, the controller now requires more time to achieve consensus. The profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ can be seen in Fig 4.9. Similar to the previous scenario, $\hat{w}_i$ is not driven to w_i, but to $w_i + \rho_i(v_i)$ for deliberately designed $\rho_i(v_i)$. The profile of z_i is plotted in Fig. 4.10.

Based on illustrated simulation results, we can verify that asymptotic consensus can be achieved using our approach as concluded in Theorem 4.4.1. We also can see the effectiveness of the proposed adaptive controller to handle uncertain nonlinearities.

4.6 Summary

In this chapter, we have presented a distributed adaptive scheme for a MAS that aims to maintain its nominal collective behaviour subject to uncertain nonlinearities. The main idea is to drive the estimation error to a deliberately designed manifold in the

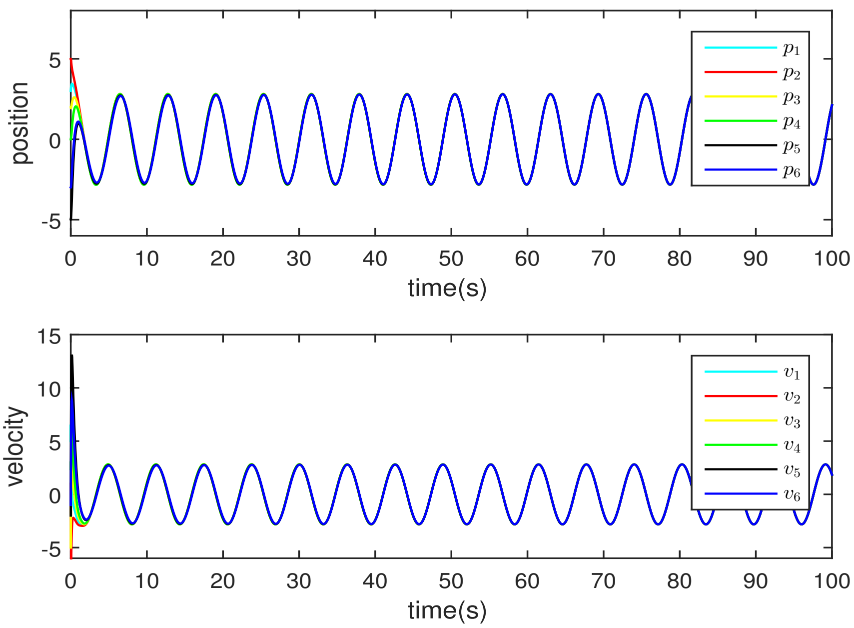

Figure 4.5: Profile of state trajectories of six agents with adaptive controller

space of agent states and estimated parameters, which provides significant advantages in distributed implementation compared with the traditional adaptive law based on gradient of a Lyapunov function. The effectiveness of the new scheme has been demonstrated in solving an open asymptotic consensus problem for a second-order MAS in a leaderless directed network.

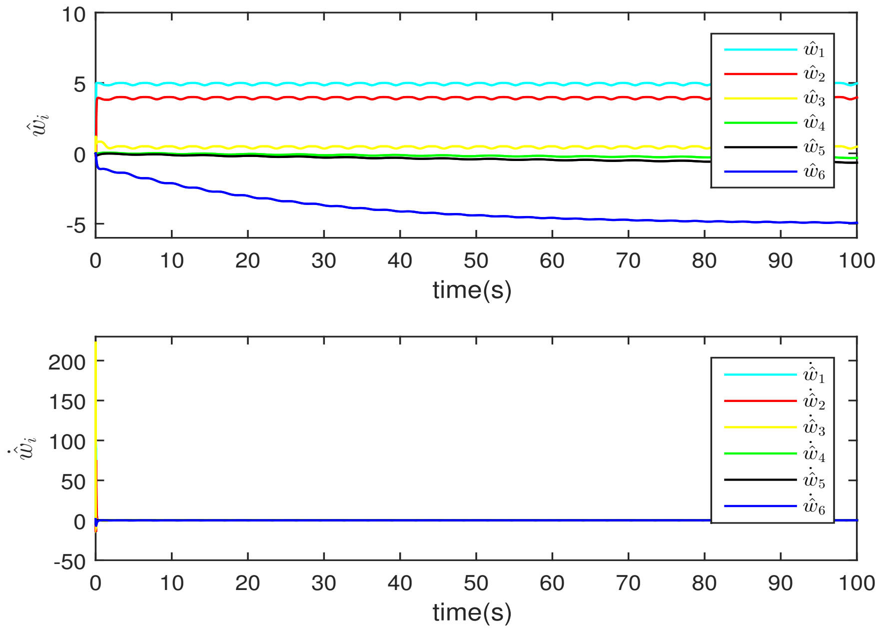

Figure 4.6: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ of six agents with adaptive controller

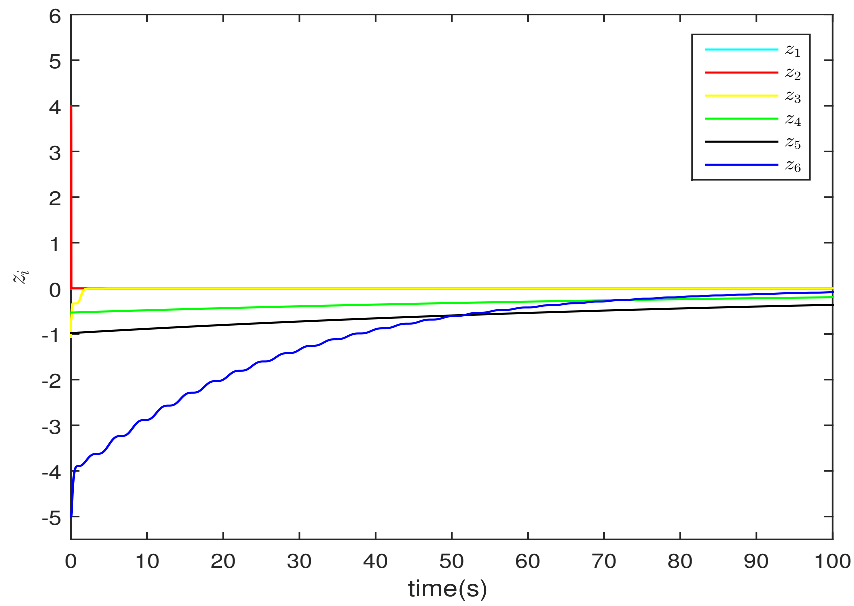

Figure 4.7: Profile of z_i of six agents with adaptive controller

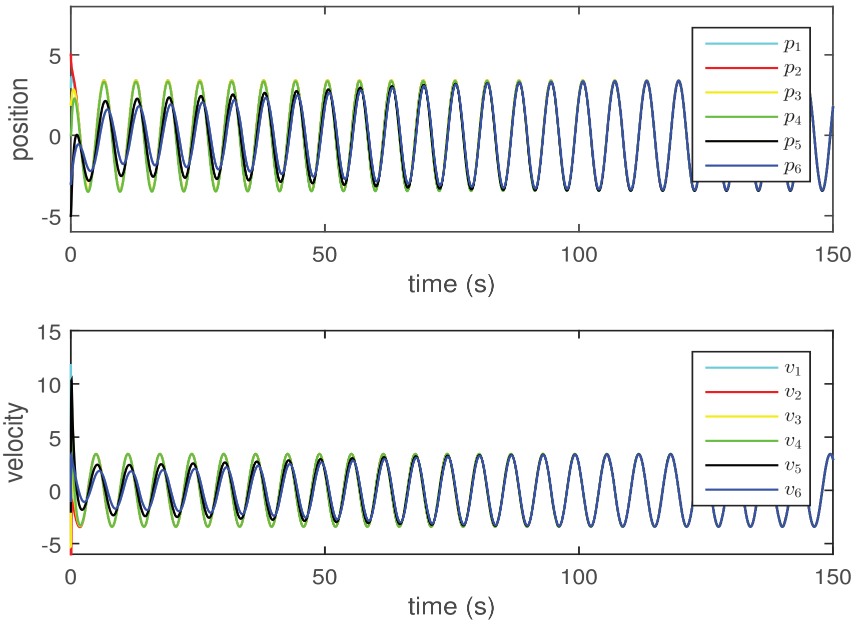

Figure 4.8: Profile of state trajectories of six agents with unknown constant parameters within interval $[-50, 50]$

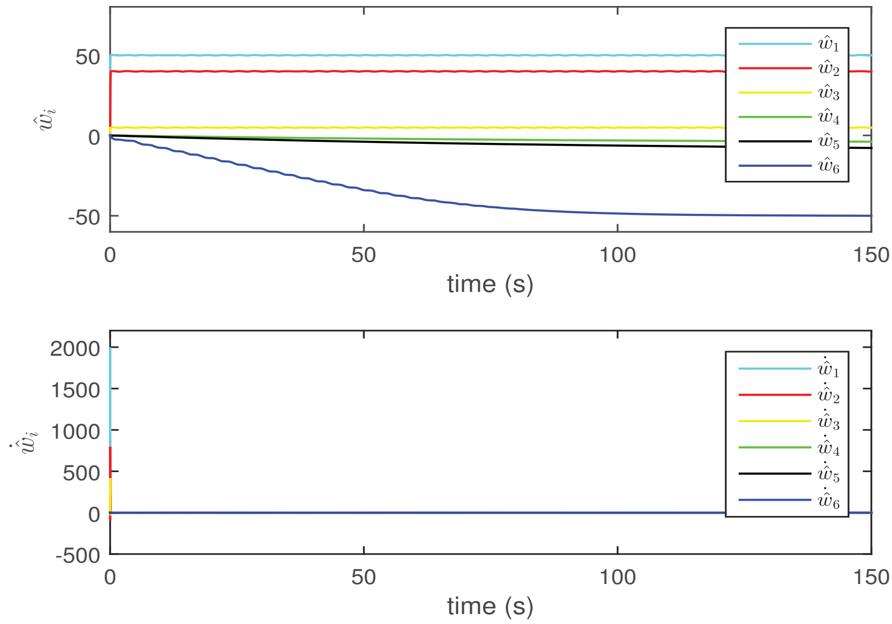

Figure 4.9: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ of six agents with unknown constant parameters within interval $[-50, 50]$

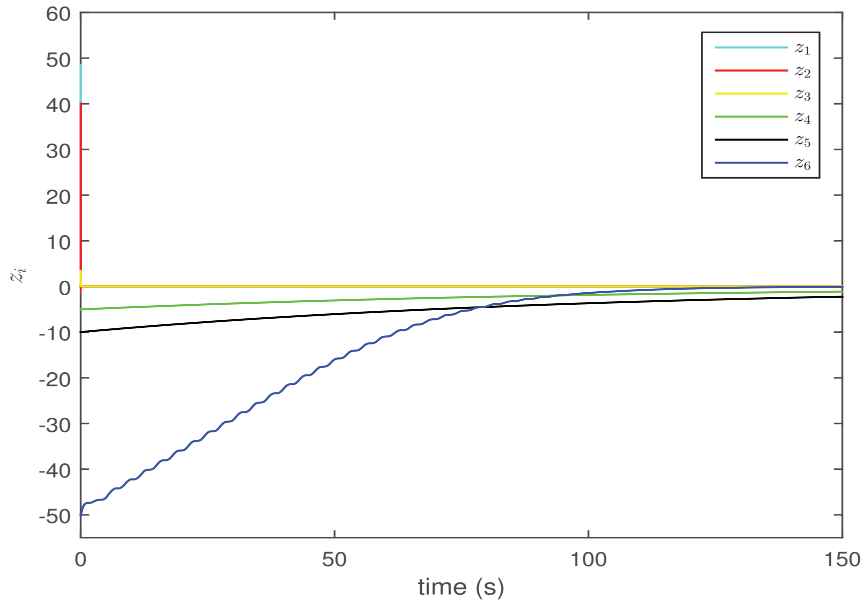

Figure 4.10: Profile of z_i of six agents with unknown constant parameters within interval $[-50, 50]$

5

Distributed Adaptive Consensus of Multi-Agent Systems with Nonlinearly Parameterized Dynamics

5.1 Introduction

Handling nonlinearly parameterized uncertainties is always a difficult issue in adaptive control even for a single system scenario. There are some existing results as follows. The research based on convex/concave nonlinear functions is one of the major research lines for adaptive control of nonlinearly parameterized models, see, e.g. [109, 110, 111, 112]. In [113], the adaptive control for nonlinearly parameterized systems was proposed by exploiting the monotonicity property of nonlinear functions. In [115], the so-called immersion and invariance adaptive control was proposed by constructing a monotone mapping. An adaptive control method for the class of strict-feedback nonlinearly parameterized systems was studied in [119] by introducing a biasing vector function into parameter estimation. Another novel adaptive control approach based on forward/backward adaptation law was established to achieve system stability and parameter convergence in [116] that does not rely on the explicit expression of system nonlinearities.

However, none of these results have been successfully applied in a networked setting.

In this chapter, we extend the consensus framework in Chapter 4 to be more general, where the dynamics of MASs contain nonlinearly parameterized uncertainties. This is the first time to pursue a distributed adaptive consensus controller for nonlinearly parameterized systems. The linear parameterization assumption will be removed by a novel distributed adaptive update law, which makes the scheme applicable for more general nonlinear MASs. In this scheme, the adaptive estimation error is driven to a deliberately designed manifold in the space of agent states and estimated parameters. The new adaptive scheme will be designed for more general nonlinearly parameterized uncertainties. With this new scheme, we will be able to solve an open consensus problem for a leaderless second-order MAS under a directed network.

The remainder of this chapter is organized as follows. We present the problem formulation and some preliminary results in Section 5.2. In Section 5.3, a new distributed adaptive scheme is proposed for MASs to maintain their nominal behaviour subject to more general uncertain nonlinearities. An open problem of adaptive consensus for second-order MAS is solved in Section 5.4. To illustrate the effectiveness of our design, a numerical example is given in Section 5.5. Finally, the chapter is closed with some concluding remarks in Section 5.6.

5.2 Problem Formulation and Preliminaries

Recall a MAS of n agents under properly designed controllers (3.1) represented by

$$\dot{x}_i = f_i(x),\ i = 1, \cdots, n \tag{5.1}$$

where $x_i \in \mathbb{R}^l$ is the state of the i-th agent and $x = [x_1^\intercal, x_2^\intercal, \cdots, x_n^\intercal]^\intercal$. The dependence of the function f_i on x (not only x_i) means the interconnection among agents. Let $f(x) = [f_1^\intercal(x), f_2^\intercal(x), \cdots, f_n^\intercal(x)]^\intercal$. Then, the nominal uncertainty-free MAS (5.1) can be put in a compact form as $\dot{x} = f(x)$. Suppose the MAS has achieved a certain collective behaviour, specifically, with a property in terms of a Lyapunov-like function. That is, Assumption 4.2.1 is satisfied. Two typical selections of R in Assumption 4.2.1 is described in Remark 4.2.1.

The research target is to propose an adaptive law in a superposition form on top of the existing controller, such that the behaviour of the nominal system is still maintained when the system is subject to uncertainties. Validity of such a superposition design is called the certainty equivalence principle. Specifically, the uncertainties and the additional adaptive control are taken into the system (5.1) in the manner described by

$$\dot{x}_i = f_i(x) + f_i^{\delta}(x_i, w_i, \mu_i), \; i = 1, \cdots, n \tag{5.2}$$

where $w_i \in \mathbb{R}^{s_i}$ denotes the unknown constant parameters and $\mu_i \in \mathbb{R}^{s_i}$ the additional adaptive control input to handle these uncertainties. It is assumed that $\mu_i = w_i$ could cancel the uncertainties if the parameter w_i were known, that is,

$$f_i^{\delta}(x_i, w_i, \mu_i) := g_i(x_i, w_i) - g_i(x_i, \mu_i)$$

for some function g_i, throughout this chapter. In the practical scenario that w_i is unknown, an adaptive law is required for μ_i to dynamically cancel the uncertainties associated with the parameter w_i.

There are two major challenges in designing a distributed adaptive law in the present networked setting. First, an adaptive law usually depends on the gradient of the established Lyapunov function for the nominal system, which is $V(x)$ in Assumption 4.2.1 for the present case. More specifically, the adaptive law depends on $\partial V(x)/\partial x_i$ for each agent i, which, hence, depends on not only the local state of agent i, but also the full network state x. This is an obstacle for a distributed adaptive law. This challenge has been overcome in Chapter 4 for the special case under the linearly parameterized constraint, i.e.,

$$g_i(x_i, w_i) = h_i(x_i)w_i, \; g_i(x_i, \mu_i) = h_i(x_i)\mu_i \tag{5.3}$$

for some function $h_i(x_i)$. The main result is stated in Theorem 4.3.1.

The second challenge is to handle more general nonlinearly parameterized uncertainties, which is always a difficult issue in adaptive control even for an individual (non-networked) scenario. Together with the first challenge, the question is how to remove the linearly parameterized constraint (5.3) in Theorem 4.3.1. This is not a trivial extension. Without the constraint (5.3), the function $h_i(x_i)$ required for the adaptive controller (4.10) does not exist. In the next section, we will find a strategy to construct a suitable $h_i(x_i)$ that plays the same role, but is capable of tackling nonlinearly parameterized uncertainties.

5.3 A Distributed Adaptive Scheme

The main result of this section is to give the explicit condition for $h_i(x_i)$ and hence (4.10) and (4.11) for nonlinearly parameterized systems such that Theorem 4.3.1 still holds without the constraint (5.3). It is noted that for a linearly parameterized $g_i(x_i, w_i)$, the following property holds for any vector z_i of the same dimension of w_i,

$$g_i(x_i, w_i) - g_i(x_i, w_i - z_i) = g_i(x_i, z_i) = h_i(x_i)z_i, \tag{5.4}$$

where

$$z_i = \beta_i(x_i) - \tilde{w}_i, \ \tilde{w}_i = \hat{w}_i - w_i. \tag{5.5}$$

When $g_i(x_i, w_i)$ is nonlinearly parameterized, we will construct two functions $h_i(x_i)$ and $\bar{\tau}_i(z_i)$ satisfying the following condition

$$\begin{aligned}[][h_i(x_i)\bar{\tau}_i(z_i) - g_i(x_i, w_i) + g_i(x_i, w_i - z_i)]^\mathsf{T} \\ \cdot [g_i(x_i, w_i) - g_i(x_i, w_i - z_i)] \geq 0,\end{aligned} \tag{5.6}$$

Remark 5.3.1 The condition (5.6) has two-fold meanings. On one hand, $g_i(x_i, w_i) - g_i(x_i, w_i - z_i)$ represents the change direction of the function $g_i(x_i, w_i)$ along parameter w_i. The selection of $h_i(x_i)\bar{\tau}_i(z_i)$ is along the change direction in the sense of

$$[h_i(x_i)\bar{\tau}_i(z_i)]^\mathsf{T} \cdot [g_i(x_i, w_i) - g_i(x_i, w_i - z_i)] \geq 0. \tag{5.7}$$

On the other hand, $h_i(x_i)\bar{\tau}_i(z_i)$ determines the boundary of $g_i(x_i, w_i) - g_i(x_i, w_i - z_i)$ in the sense of

$$\begin{aligned}\|[h_i(x_i)\bar{\tau}_i(z_i)]^\mathsf{T} \cdot [g_i(x_i, w_i) - g_i(x_i, w_i - z_i)]\| \\ \geq \|g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\|^2,\end{aligned} \tag{5.8}$$

that requires

$$\|h_i(x_i)\bar{\tau}_i(z_i)\| \cdot \|g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\|. \tag{5.9}$$

It is easy to verify that (5.6) is equivalent to (5.7)+(5.8). For many classes of functions g_i, the two functions $h_i(x_i)$ and $\bar{\tau}_i(z_i)$ can be constructed to satisfy (5.7) and (5.8), without knowing the exact value of w_i. An example is given in Section 5.4. ∎

The role of condition (5.6) will be seen in the main result summarized in the following theorem.

Theorem 5.3.1 *Consider the system (5.2) under Assumption 4.2.1. Suppose there exist two functions $h_i(x_i)$ and $\varrho_i(\cdot) > 0$ and a continuously differentiable positive definite function $W_i(z_i)$ such that (5.6) holds for*

$$\bar{\tau}_i(z_i) = \varrho_i(W_i(z_i))(\frac{\partial W_i(z_i)}{\partial z_i})^{\mathsf{T}}. \tag{5.10}$$

Let the distributed adaptive controller be

$$\begin{aligned} \mu_i &= \hat{w}_i - \beta_i(x_i) \\ \dot{\hat{w}}_i &= -\lambda_i h_i^{\mathsf{T}}(x_i) f_i(x) \end{aligned} \tag{5.11}$$

where $\beta_i(x_i)$ is any continuously differentiable function satisfying

$$\frac{\partial \beta_i(x_i)}{\partial x_i} = -\lambda_i h_i^{\mathsf{T}}(x_i), \tag{5.12}$$

for some $\lambda_i > 0$. Then, the derivative of

$$U(x,z) = V(x) + \frac{\sigma}{4(1-k)} \sum_{i=1}^{n} \int_0^{W_i(z_i)} \frac{\varrho_i(s)}{\lambda_i} ds \tag{5.13}$$

with (5.5) and $z = [z_1^{\mathsf{T}}, z_2^{\mathsf{T}}, \cdots, z_n^{\mathsf{T}}]^{\mathsf{T}}$ satisfies

$$\dot{U}(x,z) \le -k\alpha(\|x\|_R), \tag{5.14}$$

for any $0 < k < 1$, along the trajectory of the closed-loop system (5.2)+(5.11). ∎

Proof: We can rewrite the closed-loop system composed of (5.2) and (5.11) as follows, noting $\mu_i = w_i - z_i$,

$$\dot{x}_i = f_i(x) + g_i(x_i, w_i) - g_i(x_i, w_i - z_i). \tag{5.15}$$

By direct calculation, we have the time-derivative of the function $V(x)$, along the trajectory of (5.15), as follows

$$\begin{aligned} \dot{V}(x) &= \frac{\partial V(x)}{\partial x} f(x) + \sum_{i=1}^{n} \frac{\partial V(x)}{\partial x_i} \left(g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\right) \\ &\le -\alpha(\|x\|_R) + \sum_{i=1}^{n} \frac{\partial V(x)}{\partial x_i} \left(g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\right). \end{aligned}$$

For any $0 < k < 1$, pick $a = (1-k)/\sigma$. Then,

$$a \left\| \frac{\partial V(x)}{\partial x} \right\|^2 \le (1-k)\alpha(\|x\|_R). \tag{5.16}$$

Furthermore,

$$\begin{aligned}\dot{V}(x) &\leq -\alpha(\|x\|_R) + \sum_{i=1}^{n} a \left\| \frac{\partial V(x)}{\partial x_i} \right\|^2 \\ &\quad + \frac{1}{4a} \sum_{i=1}^{n} \|g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\|^2 \\ &\leq -\alpha(\|x\|_R) + a \left\| \frac{\partial V(x)}{\partial x} \right\|^2 \\ &\quad + \frac{1}{4a} \sum_{i=1}^{n} \|g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\|^2 \\ &\leq -k\alpha(\|x\|_R) + \frac{1}{4a} \sum_{i=1}^{n} \|g_i(x_i, w_i) - g_i(x_i, w_i - z_i)\|^2 .\end{aligned}$$

Next, noting that

$$\dot{\hat{w}}_i = \frac{\partial \beta_i(x_i)}{\partial x_i} f_i(x), \tag{5.17}$$

the dynamics of z_i can be written as

$$\begin{aligned}\dot{z}_i =& \frac{\partial \beta_i(x_i)}{\partial x_i} \dot{x}_i - \dot{\hat{w}}_i \\ =& \frac{\partial \beta_i(x_i)}{\partial x_i} \left(f_i(x) + g_i(x_i, w_i) - g_i(x_i, w_i - z_i) \right) - \frac{\partial \beta_i(x_i)}{\partial x_i} f_i\left(x\right) \\ =& \frac{\partial \beta_i(x_i)}{\partial x_i} \left(g_i(x_i, w_i) - g_i(x_i, w_i - z_i) \right).\end{aligned} \tag{5.18}$$

Therefore, we can verify that the time-derivative of $W_i(z_i)$, along the trajectory of the aforementioned z_i-dynamics, satisfies

$$\dot{W}_i(z_i) = \frac{\partial W_i(z_i)}{\partial z_i} \frac{\partial \beta_i(x_i)}{\partial x_i} \left(g_i(x_i, w_i) - g_i(x_i, w_i - z_i) \right). \tag{5.19}$$

From above, the time-derivative of $U(x, z)$, along the trajectory of the closed-loop system, is

$$
\begin{aligned}
\dot{U}(x,z) \leq & -k\alpha(\|x\|_R) + \frac{1}{4a}\sum_{i=1}^{n}\|g_i(x_i,w_i) - g_i(x_i,w_i - z_i)\|^2 \\
& + \frac{1}{4a}\sum_{i=1}^{n}\varrho_i(W_i(z_i))\frac{\partial W_i(z_i)}{\partial z_i}\frac{\partial \beta_i(x_i)}{\partial x_i} \\
& \cdot \left(g_i(x_i,w_i) - g_i(x_i,w_i - z_i)\right)/\lambda_i \\
\leq & -k\alpha(\|x\|_R) + \frac{1}{4a}\sum_{i=1}^{n}\|g_i(x_i,w_i) - g_i(x_i,w_i - z_i)\|^2 \\
& - \frac{1}{4a}\sum_{i=1}^{n}\bar{\tau}_i^{\mathsf{T}}(z_i)h_i^{\mathsf{T}}(x_i)\left(g_i(x_i,w_i) - g_i(x_i,w_i - z_i)\right) \\
\leq & -k\alpha(\|x\|_R)
\end{aligned}
\tag{5.20}
$$

due to (5.6). The proof is thus completed. ■

Remark 5.3.2 With the constraint (5.3), the condition (5.6) automatically holds for $\varrho_i(\cdot) = 1$, $W_i(z_i) = z_i^{\mathsf{T}} z_i/2$, and $\bar{\tau}_i(z_i) = z_i$. Then, the function $U(x,z)$ in (5.13) reduces to that in (4.12), and hence Theorem 5.3.1 to Theorem 4.3.1. In other words, there is no additional conservativeness applied on Theorem 5.3.1, compared with its special version Theorem 4.3.1. ▮

Remark 5.3.3 When the function $g_i(x_i, w_i)$ is a general nonlinear function, it is not difficult to find two functions $h_i(x_i)$ and $\bar{\tau}_i(z_i)$ to satisfy (5.6). However, it should be noted that the existence of $\beta_i(x_i)$ satisfying (5.12) is not always guaranteed for every $h_i(x_i)$. In other words, it is challenging to find a suitable $h_i(x_i)$ for both (5.12) and (5.6) simultaneously. The solution will be found on a case-by-case basis. ▮

Remark 5.3.4 The adaptive controller (5.11) in Theorem 5.3.1 is applied to every agent i. This distributed control protocol only relies on the local agent state x_i and its nominal dynamics $f_i(x)$. The MAS with the nominal dynamics $f_i(x)$ is implemented in distributed fashion beforehand, when the nonlinear function $g_i(x_i, w_i)$ has not been taken into consideration. ▮

5.4 Application to A Network of Second-Order Uncertain Dynamics

In this section, we study adaptive consensus for a second-order MAS using the scheme in Theorem 5.3.1. Consider $n \geq 2$ autonomous agents governed by the set of equations

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= \alpha_1 p_i + \alpha_2 v_i + \xi_i(v_i, w_i) + u_i,\ i = 1, \ldots, n, \end{aligned} \tag{5.21}$$

where $p_i, v_i \in \mathbb{R}$ are the states, the constants $\alpha_1, \alpha_2 \in \mathbb{R}$ are the known parameters and $u_i \in \mathbb{R}$ is the control input of agent i and $\xi_i(v_i, w_i)$ is a bounded nonlinear function with unknown constant parameter w_i. For convenience of presentation, we define

$$A = \begin{bmatrix} 0 & 1 \\ \alpha_1 & \alpha_2 \end{bmatrix},\ x_i = \begin{bmatrix} p_i \\ v_i \end{bmatrix}$$

and

$$p = \begin{bmatrix} p_1 \\ \vdots \\ p_n \end{bmatrix}, v = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}. \tag{5.22}$$

In this section, the network topology is represented by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$. A finite non-empty set of nodes is denoted by $\mathcal{V} = \{1, 2, \cdots, n\}$ and the set of directed edges is presented by $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. Let $\mathcal{A} = [a_{ij}]$ denote the adjacency matrix where $a_{ij} > 0$ if the edge $(j, i) \in \mathcal{E}$, $i \neq j$, $a_{ij} = 0$ if $i = j$. So, there exists no self-loop. Define the Laplacian matrix as $L = [L_{ij}]$ that has elements of $L_{ij} = -a_{ij}$, $j \neq i$ and $L_{ii} = \sum_{j=1, j\neq 1}^{n} a_{ij}$. Let L_i be the i-th row of L, then the distributed information from the network to the agent i can be written as

$$\begin{aligned} L_i p &= -\sum_{j=1}^{n} a_{ij}(p_j - p_i) \\ L_i v &= -\sum_{j=1}^{n} a_{ij}(v_j - v_i). \end{aligned}$$

In this section, we investigate a general directed leaderless MAS with the following assumption.

Assumption 5.4.1 The network topology contains at least a directed spanning tree. ∎

Under Assumption 5.4.1, the Laplacian matrix L has one zero eigenvalue and all the remaining eigenvalues are with positive real parts. Let $r \in \mathbb{R}^n$ and $\mathbf{1}$ be the left and right eigenvectors corresponding to the eigenvalue 0. We have $r^\top L = 0$, $L\mathbf{1} = 0$, and $r^\top \mathbf{1} = 1$. There exist matrices $W \in \mathbb{R}^{(n-1)\times n}$, $U \in \mathbb{R}^{n\times(n-1)}$ such that

$$T = \begin{bmatrix} r^\top \\ W \end{bmatrix}, \ T^{-1} = \begin{bmatrix} \mathbf{1} & U \end{bmatrix}. \tag{5.23}$$

Then, the Laplacian matrix L can be transformed into

$$TLT^{-1} = \begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix}, \tag{5.24}$$

where $J = WLU \in \mathbb{R}^{(n-1)\times(n-1)}$ is the matrix with positive eigenvalues of L on the diagonal. Let us define the matrix R as follows

$$\begin{bmatrix} Wp \\ Wv \end{bmatrix} = Rx \tag{5.25}$$

where R has a full row rank and the rows of R are perpendicular to span $\{\mathbf{1} \otimes I_2\}$. There are two technical lemmas regarding the property of the nominal system (5.21) with $\xi_i(v_i, w_i) = 0$.

In this chapter, we will use two technical Lemmas, i.e. Lemma 4.4.1 and 4.4.2 presented in Chapter 4. According to Lemma 4.4.1, by selecting sufficiently large γ_1 and $\gamma_2 = c\gamma_1 > 1$ with $c > 0$, then the matrix

$$\bar{A} = \begin{bmatrix} 0 & I \\ \alpha_1 I - \gamma_1 J & \alpha_2 I - \gamma_2 J \end{bmatrix}$$

is Hurwitz. From Lemma 4.4.2, we can conclude that MAS (5.21) with $\xi_i(v_i, w_i) = 0$ under Assumption 5.4.1 achieves consensus under the following control protocol

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v, \tag{5.26}$$

where γ_1 and γ_2 are properly selected such that the matrix $\bar{A}$ is Hurwitz.

Now, the main result on a distributed adaptive controller for the MAS (5.21) is stated in the following theorem.

Theorem 5.4.1 *Consider the MAS (5.21) under Assumption 5.4.1. Suppose there exist two functions* $\hbar_i(v_i)$ *and* $\varrho_i(\cdot) > 0$ *such that, with* $\bar{\tau}_i(\varsigma_i) = \varrho_i(\varsigma_i^2/2)\varsigma_i$,

$$\begin{aligned}&[\hbar_i(v_i)\bar{\tau}_i(\varsigma_i) - \xi_i(v_i, w_i) + \xi_i(v_i, w_i - \varsigma_i)]^\intercal \\ &\qquad \cdot [\xi_i(v_i, w_i) - \xi_i(v_i, w_i - \varsigma_i)] \geq 0. \end{aligned} \tag{5.27}$$

Let the controller be

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v - \xi_i(v_i, \mu_i) \tag{5.28}$$

and

$$\begin{aligned}\mu_i &= \hat{w}_i - \rho_i(v_i) \\ \dot{\hat{w}}_i &= -\lambda_i \hbar_i^\intercal(v_i)[\alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v], \end{aligned} \tag{5.29}$$

where γ_1 *and* γ_2 *are selected according to Lemma 4.4.2 and* $\rho_i(v_i)$ *is a continuously differentiable function satisfying*

$$\frac{\partial \rho_i(v_i)}{\partial v_i} = -\lambda_i \hbar_i^\intercal(v_i) \tag{5.30}$$

for some $\lambda_i > 0$. *Then, the closed-loop system (5.21)+(5.28)+(5.29) achieves consensus in the sense of*

$$\begin{aligned}\lim_{t\to\infty} p_i(t) - p_o(t) =& 0 \\ \lim_{t\to\infty} v_i(t) - v_o(t) =& 0 \end{aligned} \tag{5.31}$$

for some functions $p_o(t), v_o(t) : [0, \infty) \mapsto \mathbb{R}$. ∎

Proof: First of all, the system composed of (5.21) and (5.28) can be written as follows,

$$\begin{aligned}\dot{p}_i =& v_i \\ \dot{v}_i =& \alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v + \xi_i(v_i, w_i) - \xi_i(v_i, \mu_i), \ i = 1, \cdots, n. \end{aligned}$$

It can also be put in the following compact form,

$$\dot{x}_i = f_i(x) + g_i(x_i, w_i) - g_i(x_i, \mu_i) \tag{5.32}$$

where the nominal dynamics $\dot{x}_i = f_i(x)$ is given by

$$\begin{aligned}\dot{p}_i &= v_i \\ \dot{v}_i &= \alpha_1 p_i + \alpha_2 v_i - \gamma_1 L_i p - \gamma_2 L_i v, \ i = 1, \cdots, n \end{aligned} \tag{5.33}$$

and

$$\begin{aligned} g_i(x_i, w_i) &= \begin{bmatrix} 0 \\ \xi_i(v_i, w_i) \end{bmatrix} \\ g_i(x_i, \mu_i) &= \begin{bmatrix} 0 \\ \xi_i(v_i, \mu_i) \end{bmatrix}. \end{aligned} \tag{5.34}$$

The system (5.32) takes the form (5.2). Applying Lemma 4.4.2 verifies Assumption 4.2.1 for $\dot{x}_i = f_i(x)$. Furthermore, we can verify that

$$\frac{\left\| \frac{\partial V(x)}{\partial x} \right\|^2}{\|x\|_R^2} = \frac{\|2x^\top R^\top P R\|^2}{\|x\|_R^2} \leq 4\|PR\|^2 < \infty. \tag{5.35}$$

By Theorem 5.3.1 with $\varsigma_i = \rho_i(v_i) - \hat{w}_i + w_i$, $h(x_i) = \hbar_i(v_i)$, $\bar{\tau}_i(z_i) = \bar{\tau}_i(\varsigma_i)$, $\beta_i(x_i) = \rho_i(v_i)$, and

$$U(x, \varsigma) = V(x) + \frac{\sigma}{4(1-k)} \sum_{i=1}^{n} \int_0^{\varsigma_i^2/2} \frac{\varrho_i(s)}{\lambda_i} ds, \tag{5.36}$$

one has

$$\dot{U}(x, \varsigma) \leq -k\|x\|_R^2. \tag{5.37}$$

It is noted that $U(x(t), \varsigma(t))$ and hence $\|x(t)\|_R$ are bounded. Because of

$$R\dot{x} = \bar{A}Rx + R(g(x, w) - g(x, w - \varsigma)),$$

with $w = [w_1^\top, \cdots, w_n^\top]^\top$, $\varsigma = [\varsigma_1^\top, \cdots, \varsigma_n^\top]^\top$, and $g(x, w) = [g_1^\top(x_1, w_1), \cdots, g_n^\top(x_n, w_n)]^\top$, $\|\dot{x}(t)\|_R$ is bounded and hence $-k\|x(t)\|_R^2$ is uniformly continuous in t. Also, a finite limit

$$\lim_{t\to\infty} \int_0^t -k\|x(t)\|_R^2 \geq \lim_{t\to\infty} \int_0^t \dot{U}(x(t), \varsigma(t)) \geq -U(x(0), \varsigma(0))$$

exists. By Barbalat's Lemma, one has $\lim_{t\to\infty} \|x(t)\|_R = 0$, that is,

$$\lim_{t\to\infty} \begin{bmatrix} Wp(t) \\ Wv(t) \end{bmatrix} = 0. \tag{5.38}$$

Let $p_o(t) = r^\top p(t)$ and $v_o(t) = r^\top v(t)$. From the following relationships

$$\begin{aligned} p &= \begin{bmatrix} \mathbf{1} & U \end{bmatrix} \begin{bmatrix} r^\top p \\ Wp \end{bmatrix} \\ &= \mathbf{1}(r^\top p) + U(Wp) \end{aligned}$$

and

$$\begin{aligned} v &= \begin{bmatrix} \mathbf{1} & U \end{bmatrix} \begin{bmatrix} r^\mathsf{T} v \\ W v \end{bmatrix} \\ &= \mathbf{1}(r^\mathsf{T} v) + U(Wv) \end{aligned}$$

one has

$$\begin{aligned} \lim_{t\to\infty} p(t) - p_o(t)\mathbf{1} = U \lim_{t\to\infty} Wp(t) = 0 \\ \lim_{t\to\infty} v(t) - v_o(t)\mathbf{1} = U \lim_{t\to\infty} Wv(t) = 0. \end{aligned}$$

This completes the proof. ■

Remark 5.4.1 If parameter w_i in (5.21) were known, the following controller

$$u_i = -\gamma_1 L_i p - \gamma_2 L_i v - \xi_i(v_i, w_i) \tag{5.39}$$

could be designed to achieve consensus by directly cancelling $\xi_i(v_i, w_i)$ in (5.21). In the practical case, with w_i unknown, the real controller takes the form (5.28), which is equivalent to (5.39) with w_i replaced by its estimation μ_i. Also, the estimation is determined by the adaptive law (5.29). The design approach in Theorem 5.4.1 constitutes the certainty equivalence principle. ▮

Remark 5.4.2 The distributed adaptive control protocol (5.28) is composed of two parts. The first part is the controller (5.26) proposed to achieve consensus for the ideal case when the nonlinear function $\xi_i(v_i, w_i)$ vanishes. The second part is the adaptive controller (5.29). This additional controller is added when nonlinear function with uncertainty $\xi_i(v_i, w_i)$ is taken into account. These two parts can be designed separately as stated in Theorem 5.4.1, which constitutes the certainty equivalence principle. ▮

5.5 Numerical Simulation

We consider the following six-agent systems

$$\begin{aligned} \dot{p}_i =& v_i \\ \dot{v}_i =& -p_i + \xi_i(v_i, w_i) + u_i, \; i = 1, \dots, 6. \end{aligned} \tag{5.40}$$

The nonlinear functions $\xi_i(v_i, w_i)$ are given as follows

$$\xi_i(v_i, w_i) = \begin{cases} \tanh(w_i v_i^2), & i = 1, 2, 3, 4 \\ v_i \tanh(w_i v_i^2), & i = 5, 6, \end{cases} \tag{5.41}$$

and the unknown constant parameters w_i are selected within interval $[-1, 1]$. The communication network for the MAS (5.40) has a fixed topology, as shown in Fig. 5.1. In particular, the Laplacian matrix L of the network in Fig. 5.1 is represented by

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & -1 & 0 & 0 & 2 \end{bmatrix}. \tag{5.42}$$

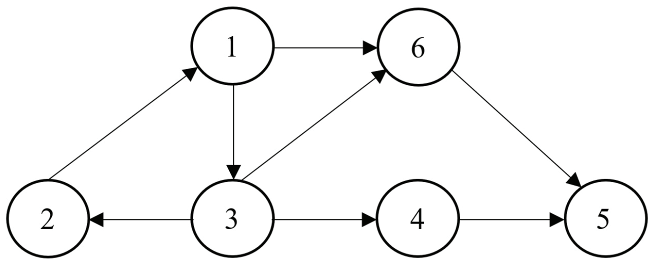

Figure 5.1: The network topology of the MAS

In this section, we demonstrate the performance of the proposed controller with several scenarios. In this simulation, we select the initial conditions

$$p(0) = \begin{bmatrix} 1 & 2 & 1.5 & 0 & -2 & -1.5 \end{bmatrix}^\top$$
$$v(0) = \begin{bmatrix} 2 & 1 & -2 & 1.5 & -2 & -1 \end{bmatrix}^\top,$$

and the unknown constant parameters

$$w = \begin{bmatrix} 1 & 0.8 & 0.6 & -1 & -0.2 & -0.3 \end{bmatrix}^\top.$$

First, the simulation is performed for MAS (5.40) without control input i.e. $u_i = 0$. Under this condition, the relative position and velocity as well as the agent states are unavailable for feedback control for any agent. Consensus is not achieved under this situation as plotted in Fig. 5.2. We can see that each agent moves from its initial position and velocity according to its own nominal dynamics and nonlinear uncertainties.

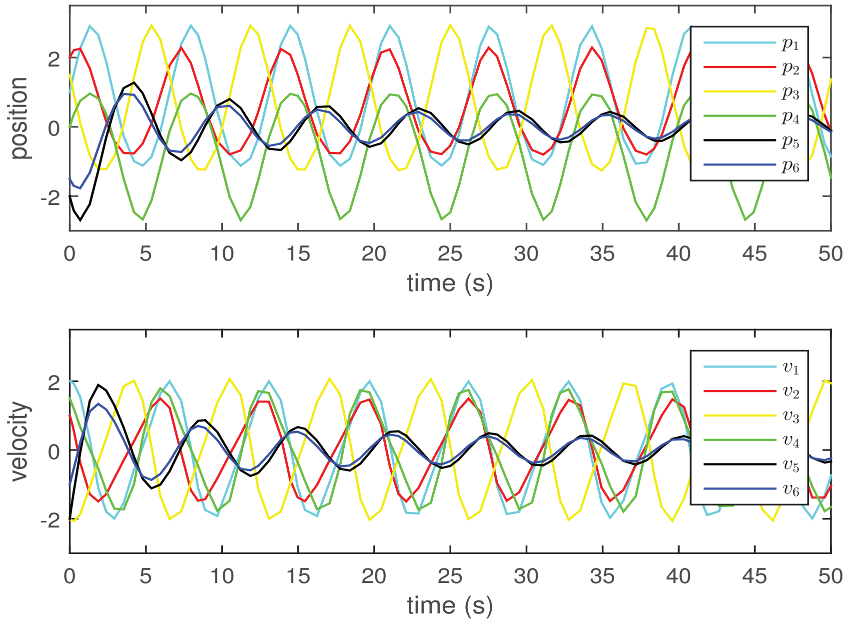

Figure 5.2: Profile of state trajectories of six agents without controller

In the second scenario, the uncertain nonlinearities in MAS (5.40) are considered vanishing i.e. $\xi_i(w_i, v_i) = 0$. By Lemma 4.4.1, we can select $\gamma_1 = 5$ and $\gamma_2 = 5$, therefore $\bar{A}$ is Hurwitz. According to Lemma 4.4.2, consensus with a collective nominal dynamics (5.40) is guaranteed to be achieved under controller (5.26). This represents MAS under ideal condition $\dot{x} = f(x)$. It is observed in Fig. 5.3 that the states of the six agents achieve consensus on a sinusoidal trajectory determined by the nominal dynamics

$$\begin{aligned} \dot{p}_i &= v_i \\ \dot{v}_i &= -p_i. \end{aligned} \tag{5.43}$$

In the second scenario, we consider that the uncertain nonlinearities $\xi_i(w_i, v_i)$ exist in the closed-loop systems. An additional controller is needed to be added in the control structure to maintain nonlinearities. We propose the distributed consensus control (5.28) to handle this situation.

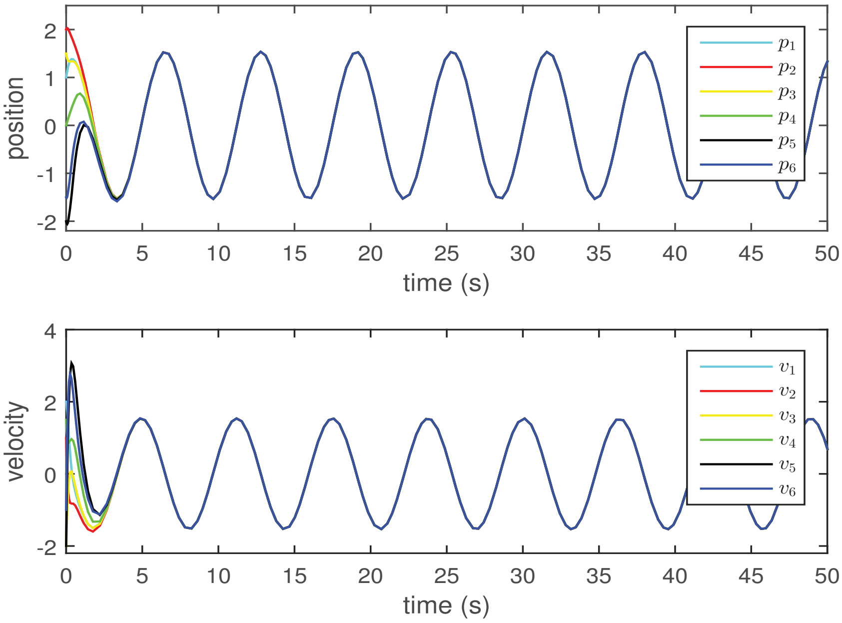

Figure 5.3: Profile of state trajectories of six agents under ideal situation

Pick the function $\hbar_i(v_i)$ as follows,

$$\hbar_i(v_i) = \begin{cases} v_i^2 + 1, & i = 1,2,3,4 \\ v_i^3 + v_i, & i = 5,6 \end{cases}. \tag{5.44}$$

First, we consider the agents $i = 1,2,3,4$. For $\varsigma_i \geq 0$, one has

$$\hbar_i(v_i)\varsigma_i \geq \varsigma_i v_i^2 \geq \tanh(w_i v_i^2) - \tanh((w_i - \varsigma_i)v_i^2) \geq 0 \tag{5.45}$$

and hence

$$\begin{aligned} &\left[\hbar_i(v_i)\varsigma_i - \tanh(w_i v_i^2) + \tanh((w_i - \varsigma_i)v_i^2)\right] \\ &\quad \cdot \left[\tanh(w_i v_i^2) - \tanh((w_i - \varsigma_i)v_i^2)\right] \geq 0, \end{aligned} \tag{5.46}$$

which verifies (5.27) with $\varrho_i(\cdot) = 1$. For $\varsigma_i \leq 0$, a similar argument follows. Next, we consider the agents $i = 5,6$. For $v_i\varsigma_i \geq 0$, one has

$$\hbar_i(v_i)\varsigma_i \geq \varsigma_i v_i^3 \geq v_i \tanh(w_i v_i^2) - v_i \tanh((w_i - \varsigma_i)v_i^2) \geq 0$$

and hence

$$\begin{aligned} &\left[\hbar_i(v_i)\varsigma_i - v_i \tanh(w_i v_i^2) + v_i \tanh((w_i - \varsigma_i)v_i^2)\right] \\ &\quad \cdot \left[v_i \tanh(w_i v_i^2) - v_i \tanh((w_i - \varsigma_i)v_i^2)\right] \geq 0, \end{aligned} \tag{5.47}$$

which verifies (5.27) with $\varrho_i(\cdot) = 1$. For $v_i \varsigma_i \leq 0$, a similar argument follows.

Now, we pick the function $\rho_i(v_i)$, satisfying (5.30) with $\lambda_i = 1$, as follows

$$\rho_i(v_i) = \begin{cases} -\frac{1}{3}v_i^3 - v_i, & i = 1,2,3,4 \\ -\frac{1}{4}v_i^4 - \frac{1}{2}v_i^2, & i = 5,6 \end{cases}. \tag{5.48}$$

As a result, the controller (5.28)+(5.29) can be explicitly constructed and Theorem 5.4.1 guarantees the achievement of consensus.

We compare the response of the closed-loop system with and without the adaptive controller to see the effectiveness of the proposed consensus control. The closed-loop system without adaptive controller is illustrated in Fig. 5.4. We can see that the states of six agents cannot achieve consensus with linear consensus control (5.26). Fig. 5.5 illustrates the profile of position and velocity of six agents to achieve consensus with adaptive controller. By adding adaptive law, the uncertain nonlinearities can be handled. The profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ are plotted in Fig. 5.6. Different to traditional adaptive controller, the state of $\hat{w}_i$ converges, not to the real value of w_i as in traditional adaptive control, but to $w_i + \rho_i(v_i)$ for a deliberately designed $\rho_i(v_i)$. For $i = 1,2,3,4$, $\rho_i(v_i)$ contains v_i and v_i^3, so $\hat{w}_i$ demonstrates the fundamental frequency 1 rad/s of v_i at the steady state. For $i = 5,6$, $\rho_i(v_i)$ contains v_i^2 and v_i^4, so $\hat{w}_i$ demonstrates the fundamental frequency 2 rad/s of v_i^2. The profile of z_i can be seen in Fig. 5.7. This simulation results show that asymptotic consensus is achieved as concluded in Theorem 5.4.1.

In the last scenario, the range of unknown constant parameters is increased to be within interval $[-50, 50]$. In this simulation, w_i is selected as follows

$$w = \begin{bmatrix} 10 & 8 & 6 & -10 & -2 & -3 \end{bmatrix}^\top.$$

Under a similar controller as the previous scenario, consensus still can be achieved by maintaining a collective nominal behaviour. Fig. 5.8 shows the profile of position and velocity consensus of six agents. Compared with the case where unknown constant parameters are within the interval $[-1, 1]$, the controller now requires more time to achieve consensus. The profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ can be seen in Fig 5.9. Similar to the previous scenario, $\hat{w}_i$ is not driven to w_i, but to $w_i + \rho_i(v_i)$ for deliberately designed $\rho_i(v_i)$. The profile of z_i is plotted in Fig. 5.10.

Based on simulation results, we can verify that asymptotic consensus can be achieved using our approach as concluded in Theorem 5.4.1. We also can see the effectiveness of the proposed adaptive controller to handle uncertain nonlinearities.

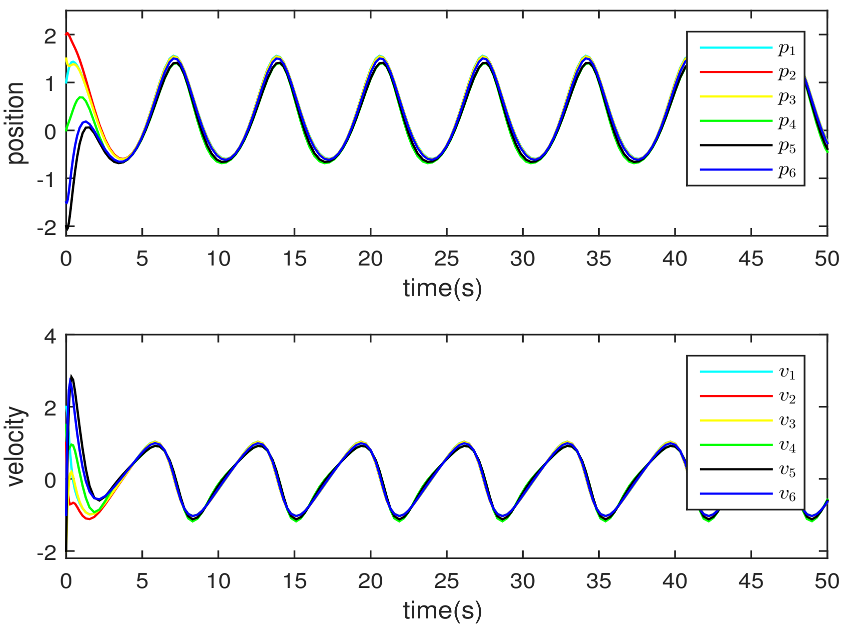

Figure 5.4: Profile of state trajectories of six agents without adaptive controller

5.6 Summary

In this chapter, we have presented a distributed adaptive consensus protocol for a MAS with uncertain nonlinearities to maintain the system's nominal collective behaviour. In this scheme, the adaptive estimation error is driven to a deliberately designed manifold in the space of agent states and estimated parameters. The new adaptive scheme is effective for general nonlinearly parameterized systems. To demonstrate the effectiveness, we have solved an open consensus problem for a leaderless second-order MAS with a directed network. It will be interesting to apply the proposed adaptive scheme for more collective control scenarios in future research.

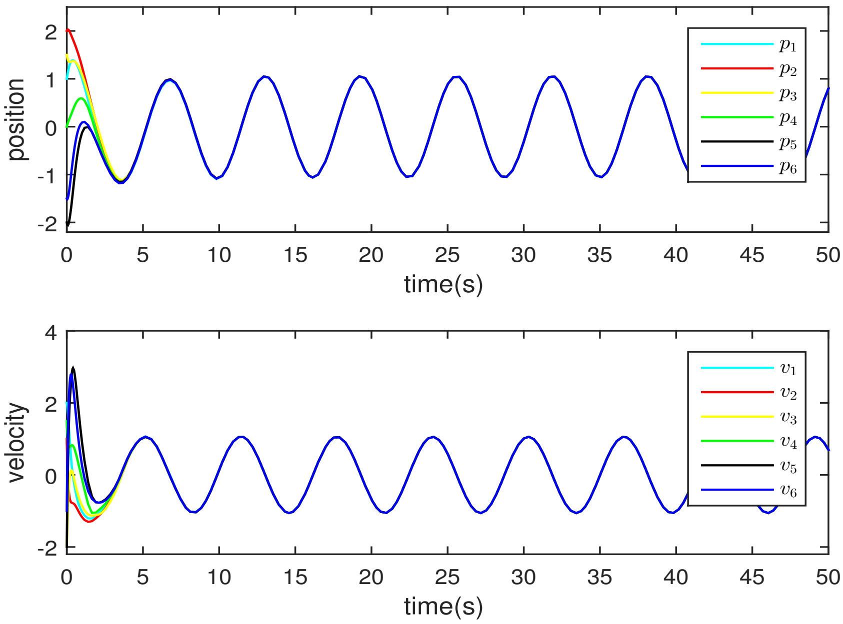

Figure 5.5: Profile of state trajectories of six agents with adaptive controller

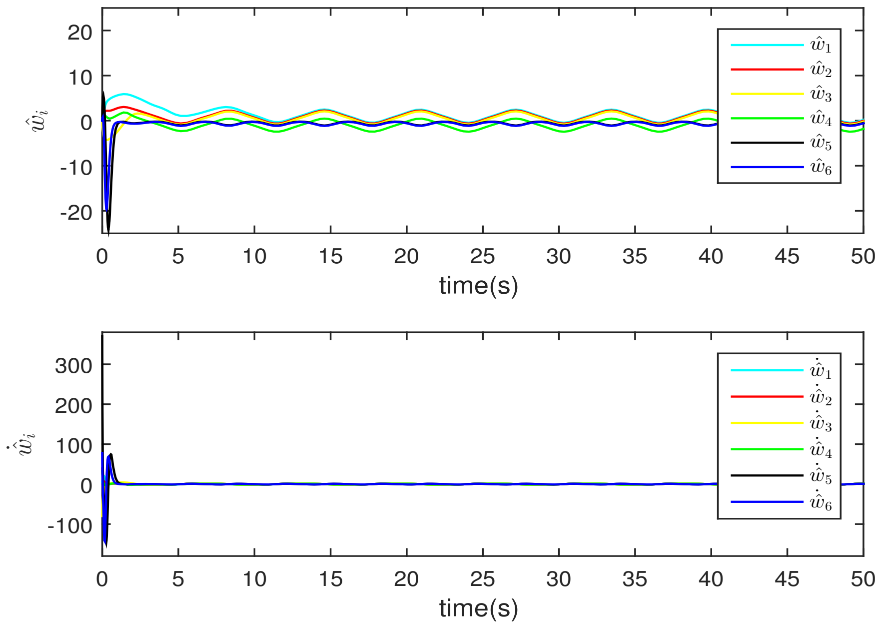

Figure 5.6: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ of six agents with adaptive controller

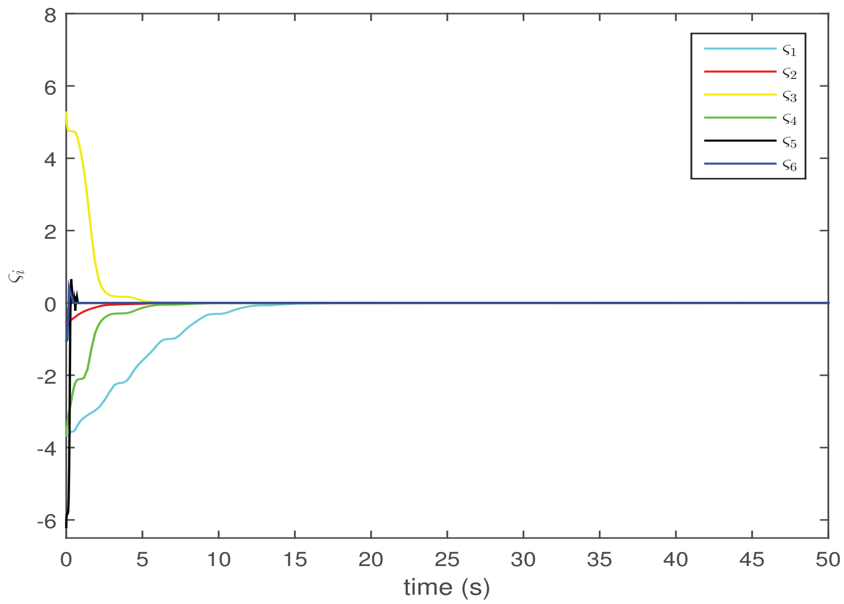

Figure 5.7: Profile of z_i of six agents with adaptive controller

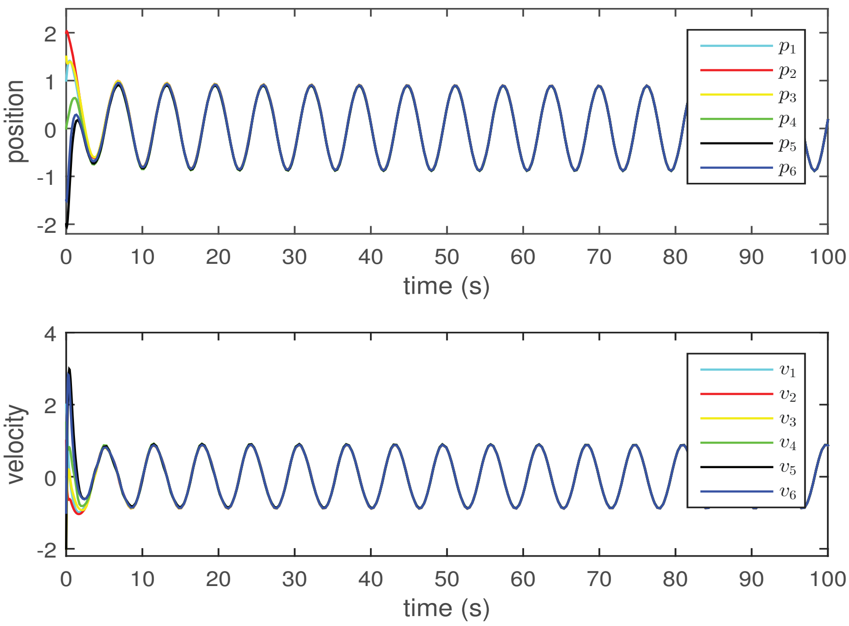

Figure 5.8: Profile of state trajectories of six agents with unknown constant parameters within interval $[-10, 10]$

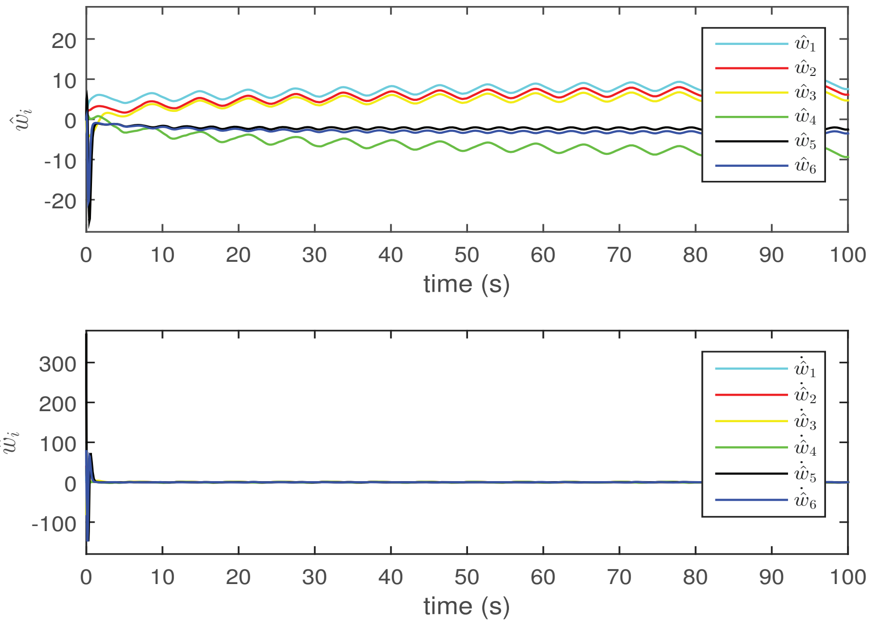

Figure 5.9: Profile of $\hat{w}_i$ and $\dot{\hat{w}}_i$ of six agents with unknown constant parameters within interval $[-10, 10]$

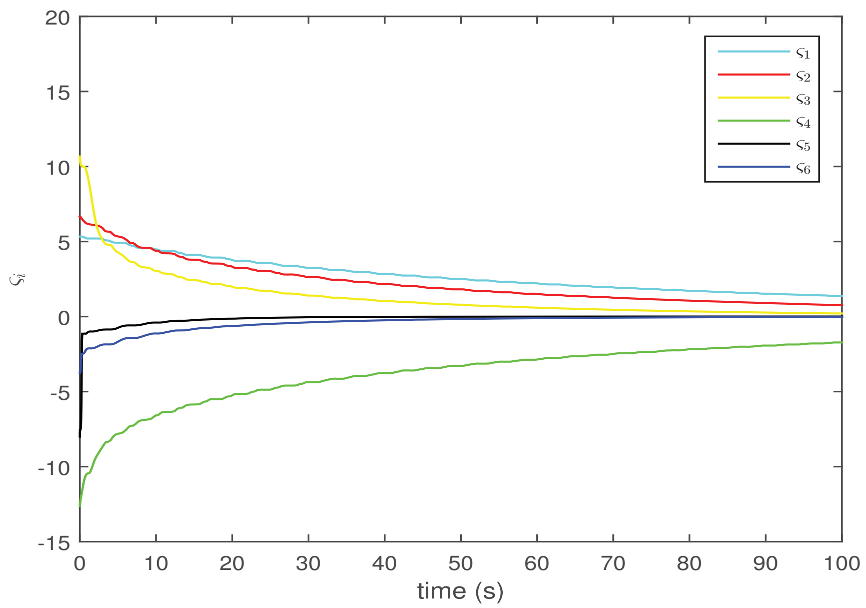

Figure 5.10: Profile of z_i of six agents with unknown constant parameters within interval $[-10, 10]$

6

Conclusion

6.1 Summary

In this thesis, our focus is to establish a distributed adaptive consensus framework for MASs subject to uncertainties. The framework can be applied for general MASs, not limited to first and second-order systems. The main results of the thesis are presented in Chapter 3, 4 and 5. Some theorems and lemmas with rigorous proofs have also been presented. To illustrate the performance of the proposed controllers, we have provided some numerical examples and simulations.

Consensus controllers are developed for both MASs with linear and nonlinear dynamics. The proposed consensus controller contains two main components. The first is a linear control protocol designed to achieve consensus with a collective nominal behaviour when the MAS is free of uncertainties. The second component is an additional adaptive compensator added in the control structure when uncertain dynamics are taken into account. The critical advantage of the proposed controller is that both components can be designed separately. The main objective is to drive all agents to achieve consensus such that the behaviour of the nominal system is still maintained.

Firstly, the introduction and research motivation to study MASs have been presented in Chapter 1. Following that, still in the same chapter, we introduce the research motivation to adaptive control. An overview of the consensus strategies for MASs with various settings such as MASs with linear dynamics, nonlinear dynamics, communication constraints and so on has been presented in the literature review section. From there, we formulate relevant research problems for consensus of MASs. Some preliminary knowledge and useful information about algebraic graph theory and adaptive control have been presented in Chapter 2.

The main results of the thesis begin in Chapter 3. A centralized adaptive scheme for a MAS that aims to maintain its nominal collective behaviour subject to uncertain nonlinearities is established in this chapter. Traditional adaptive control constituted by the certainty equivalence principle is proposed to handle uncertain dynamics. In adaptive control, the Lyapunov function is usually centrally constructed. Consequently, the global information about the network is required in designing adaptive law.

The adaptive scheme in Chapter 3 has some inherent drawbacks in designing distributed adaptive consensus control. The full network states as well as the local states are required to generate the adaptive law. The proposed approach can be implemented in a distribution fashion only for limited cases. In Chapter 4, a distributed adaptive framework is proposed for MASs subject to uncertainties. Asymptotic consensus is achieved. In this method, the estimation parameters are not driven to the actual value of unknown constant parameters, but to a deliberately designed manifold in the space of agent states and estimated parameters. An application is provided for second-order MASs with a fixed directed topology.

The proposed controller in Chapter 4 is under an essential assumption, where the uncertain nonlinear dynamics is in the class of linearly parameterized models. In Chapter 5, a general distributed adaptive framework for MASs subject to nonlinearly parameterized models is developed. We have proposed a novel distributed adaptive law to remove linearly parameterization assumption. Similar to the adaptive technique in Chapter 4, the estimation error converges to a deliberately designed manifold in the space of agent states and estimated parameters. An application to solve the leaderless consensus problem for second-order MASs with a directed network is also presented as a case study.

6.2 Outlook

In future work, it will be interesting to apply the proposed adaptive scheme to more complex control scenarios, especially for MASs with communication constraints such as time delays and switching topologies.

In real applications, time delays may be unavoidable. It will be interesting to extend our distributed adaptive scheme for MASs with time delays. Existing studies have produced results for general MASs with linear dynamics. The key issue is to generalize our adaptive law to handle nonlinearities with time delays. Firstly, this work can be started for linearly parameterized MASs. Once this consensus problem can be solved, an extension to the nonlinearly parameterized MAS can be proposed. Consensus analysis is more difficult in this situation, but the technical approach in this thesis can be generalized.

The network topology of MASs may not always be under a fixed topology. The interconnection between each agent may change over time due to hardware limitations and any possible external factors. Therefore, the existing control protocol under a fixed topology cannot be applied to handle this situation. In future work, it will be necessary to extend our proposed controller to nonlinear MASs with switching topologies, especially under jointly connected topologies.

There are two major problems for generalizing our approach with similar settings but with jointly connected topologies. The first problem is to design linear consensus control for systems with ideal situation. In our setting, the collective nominal behaviour of MASs is not always stable or marginally stable forms. Handling this situation is not trivial. Some results have been obtained in [93, 94] for MASs with jointly connected topologies, but for first-order systems. The consensus problem is more simple for first-order MASs because the velocity of unconnected agents are zero; as a result, the error position of nodes will not increase. However, the problem is more complicated for second-order systems. The velocity of unconnected agents is not zero, consequently the consensus error will increase. Some results have been obtained in [126, 127, 128] by considering velocity measurement in consensus control design. Under this situation, the switched systems consist of marginally stable subsystems. It means that the difficulty of designing a control protocol for a switched system with unstable subsystems was avoided. Another approach to maintain this situation is dynamic controller. This approach was proposed in [50, 129] without avoiding unstable subsystems in a switched system. Handling distributed static consensus for non-first-order nonlinear MASs with jointly connected topologies is challenging. Designing static consensus control contains self-contribution in the research of MASs.

The second problem is to generalize our adaptive approach for the switching systems. The adaptive control design is more complicated as it cannot be separated from linear control protocols. It will be interesting to extend the technical approach taken in this thesis to handle this situation.

6.3 Publications

The results of this thesis are based on a published article and a submitted article in specialized journals, and an accepted conference paper. The details are as below

- **Journal Papers**
  [130] Imil Hamda Imran, Zhiyong Chen, Yamin Yan, and Minyue Fu. Adaptive consensus of nonlinearly parameterized multi-agent systems. IEEE Control Systems Letters, 3(3):505-510, 2019. DOI: 10.1109/LCSYS.2019.2911688
  [131] Imil Hamda Imran, Zhiyong Chen, Lijun Zhu, and Minyue Fu. A distributed adaptive scheme for multi-agent systems. Asian Journal of Control (Submitted). https://arxiv.org/pdf/1904.11137.pdf

- **Conference Paper**
  [132] Imil Hamda Imran, Zhiyong Chen, Yamin Yan, and Minyue Fu. Adaptive consensus of nonlinearly parameterized multi-agent systems. IEEE Conference on Decision and Control 2019 (Accepted).

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